Lecture 3

Probabilistic Reliability

with

Info-Gap Uncertainty

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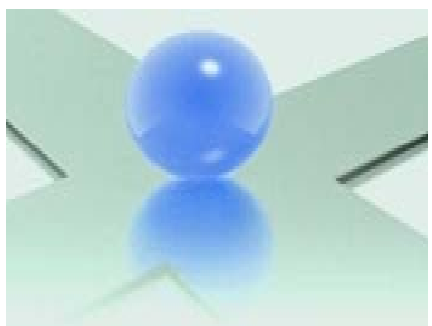

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1 *Probabilistic Reliability in Engineering with Info-Gaps*

§ Source:

Yakov Ben-Haim, *Info-Gap Economics: An Operational Introduction,* Palgrave, 2010.

1.1 The Problem: Uncertainties

§ The problem:

- **Fat tails:**
  - **Extreme outcomes too frequent.**
  - **High percentiles under-estimated.**
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  - **Data are revised.**
  - **Shackle-Popper indeterminism.**
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  - **High percentiles under-estimated.**
- **Past vs future:**
  - **Processes vary in time.**
  - **Data are revised.**
  - **Shackle-Popper indeterminism.**
- **Joint probabilities:**
  - **Uncertain common-mode failures.**
  - **Uncertain correlations.**

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- **Statistical fluctuations:**
  - **Randomness, "noise".**
  - **Estimation uncertainty.**
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  - **Historical data used to predict future.**

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§ **Two foci of uncertainty:**

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  - **Randomness, "noise".**
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§ **Info-gap theory to manage**

Knightian uncertainty.

§ Outline:

- **Discrete system with 2 sub-units:**
  - **Reliability.**
  - **Redundancy.**
  - **Uncertain correlation.**
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- **Discrete system with 2 sub-units:**
  - **Reliability.**
  - **Redundancy.**
  - **Uncertain correlation.**
- **Origin of fat tails:**

  Parameter uncertainty.
- **Quantile uncertainty.**
  - **Fat tails.**
  - **Thin tails.**

1.2 *Reliability, Redundancy, Uncertain Correlation*

§ System with 2 sub-units:

- **Serial: both units essential.**
- **Parallel: either unit sufficient.**

§

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§ **Probabilities of failure:**

- F_i = **marginal prob of failure of unit** i.
- F_{12} = **prob of joint failure.**

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- **Serial: both units essential.**
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§ Probabilities of failure:

- F_i **= marginal prob of failure of unit** i**.**
- F_{12} **= prob of joint failure.**

§ Probabilities of system failure:

- **Serial:** $F_s = F_1 + F_2 - F_{12}$.

  (Explain: Venn)
- **Parallel:** $F_p = F_{12}$

§ Example.

- **Estimates:** $\widetilde{F_i} = 0.01$. $\widetilde{F_{12}} = 0.0001$
- **Serial:** $F_{\mathrm{s}} = 0.0199$. **Not too good.**
- **Parallel:** $F_{\mathrm{p}} = 0.0001$. **Not too bad.**

§

§ Example.

- **Estimates:** $\widetilde{F}_i = 0.01$**.** $\widetilde{F}_{12} = 0.0001$
- **Serial:** $F_{\text{s}} = 0.0199$**.** **Not too good.**
- **Parallel:** $F_{\text{p}} = 0.0001$**.** **Not too bad.**

§ Problem:

Estimates of F_i **and** F_{12} **uncertain.**

$$\left|F_i - \widetilde{F}_i\right| \le h, \;\; i = 1,\, 2$$

$$\left|F_{12} - \widetilde{F}_{12}\right| \le h$$

Horizon of uncertainty, h**, is unknown.**

§ Info-gap model:

$$\begin{aligned}\mathcal{U}(h) = \{\, F_1, F_2, F_{12} : \ & F_1 \geq 0, \quad F_2 \geq 0, \quad F_{12} \geq 0. \\ & F_{12} \leq \min[F_1,\ F_2]. \\ & F_1 + F_2 - F_{12} \leq 1. \\ & \left| F_i - \widetilde{F}_i \right| \leq h,\ \ i = 1,\, 2. \\ & \left| F_{12} - \widetilde{F}_{12} \right| \leq h \,\}, \quad h \geq 0\end{aligned}$$

-

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- **Contraction:** $\mathcal{U}(0) = \{\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_{12}\}$.
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$$

- **Contraction:** $\mathcal{U}(0) = \{\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_{12}\}$.
- **Nesting:** $h < h' \implies \mathcal{U}(h) \subseteq \mathcal{U}(h')$.
- **Family of nested sets.**
-

§ Info-gap model:

$$\mathcal{U}(h) = \{\, F_1, F_2, F_{12} : \ F_1 \ge 0, \quad F_2 \ge 0, \quad F_{12} \ge 0.$$

$$F_{12} \le \min[F_1,\ F_2].$$

$$F_1 + F_2 - F_{12} \le 1.$$

$$\left|F_i - \widetilde{F}_i\right| \le h,\ \ i = 1,\, 2.$$

$$\left|F_{12} - \widetilde{F}_{12}\right| \le h \,\}\,, \qquad h \ge 0$$

- **Contraction:** $\mathcal{U}(0) = \{\widetilde{F}_1, \widetilde{F}_2, \widetilde{F}_{12}\}$.
- **Nesting:** $h < h' \implies \mathcal{U}(h) \subseteq \mathcal{U}(h')$.
- **Family of nested sets.**
- **No known worst case.**

§ Performance requirements:

- **Acceptable failure probabilities:**
  - **Serial:** $F_{\mathrm{s}} \le F_{\mathrm{cs}}$
  - **Parallel:** $F_{\mathrm{p}} \le F_{\mathrm{cp}}$

§

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§ Robustness:

- **Maximum tolerable uncertainty.**
-

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§ Performance requirements:

- **Acceptable failure probabilities:**
  - **Serial:** $F_{\rm s} \le F_{\rm cs}$
  - **Parallel:** $F_{\rm p} \le F_{\rm cp}$

§ Robustness:

- **Maximum tolerable uncertainty.**
- **Max horizon of uncertainty at which failure probability acceptable.**
- **Serial and parallel robustness:**

$$\widehat{h}_{\rm s}(F_{\rm cs}) = \max\left\{h:\ \left(\max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\rm s}\right) \le F_{\rm cs}\right\}$$

$$\widehat{h}_{\rm p}(F_{\rm cp}) = \max\left\{h:\ \left(\max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\rm p}\right) \le F_{\rm cp}\right\}$$

§ Derive parallel robustness.

§ Ramp function:

$$\overline{r}(x) = \begin{cases} 0 & x < 0 \\ x & 0 \leq x \leq 1 \\ 1 & 1 < x \end{cases} \tag{1}$$

§ Inverse of robustness:

$$\mu(h) = \max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\rm p}, \qquad F_{\rm p} = F_{12} \tag{2}$$

§

§ Derive parallel robustness.

§ Ramp function:

$$\overline{r}(x) = \begin{cases} 0 & x < 0 \\ x & 0 \le x \le 1 \\ 1 & 1 < x \end{cases} \tag{3}$$

§ Inverse of robustness:

$$\mu(h) = \max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\text{p}}, \qquad F_{\text{p}} = F_{12} \tag{4}$$

§ Uncertainty and robustness:

$$|F_{12} - \widetilde{F}_{12}| \le h \quad \Longrightarrow \quad \mu(h) = \overline{r}(\widetilde{F}_{12} + h) \tag{5}$$

$$\mu(h) \le F_{\text{cp}} \quad \Longrightarrow \quad \widehat{h}_{\text{p}}(F_{\text{cp}}) = F_{\text{cp}} - \widetilde{F}_{12} \tag{6}$$

or zero if negative.

§ Derive serial robustness.

§ Inverse of robustness:

$$\mu(h) = \max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\mathrm{s}}, \qquad F_{\mathrm{s}} = F_1 + F_2 - F_{12} \tag{7}$$

§

§ Derive serial robustness.

§ Inverse of robustness:

$$\mu(h) = \max_{F_i, F_{12} \in \mathcal{U}(h)} F_{\mathrm{s}}, \qquad F_{\mathrm{s}} = F_1 + F_2 - F_{12} \tag{8}$$

§ Uncertainty and inverse robustness:

$$F_i, F_{12} \in \mathcal{U}(h) \quad \Longrightarrow \tag{9}$$
$$\mu(h) = \overline{r}\left[\overline{r}(\widetilde{F_1} + h) + \overline{r}(\widetilde{F_2} + h) - \overline{r}(\widetilde{F_{12}} - h)\right]$$

§

§ Derive serial robustness.

§ Inverse of robustness:

$$\mu(h) = \max_{F_i, F_{12} \in \mathcal{U}(h)} F_\mathrm{s}, \qquad F_\mathrm{s} = F_1 + F_2 - F_{12} \tag{10}$$

§ Uncertainty and inverse robustness:

$$\begin{aligned} & F_i, F_{12} \in \mathcal{U}(h) \quad \Longrightarrow \\ & \mu(h) = \overline{r}\left[\overline{r}(\widetilde{F}_1 + h) + \overline{r}(\widetilde{F}_2 + h) - \overline{r}(\widetilde{F}_{12} - h)\right] \end{aligned} \tag{11}$$

§ Robustness:

- **Plot h vs. $\mu(h)$.**
- **No need to invert $\mu(h)$.**

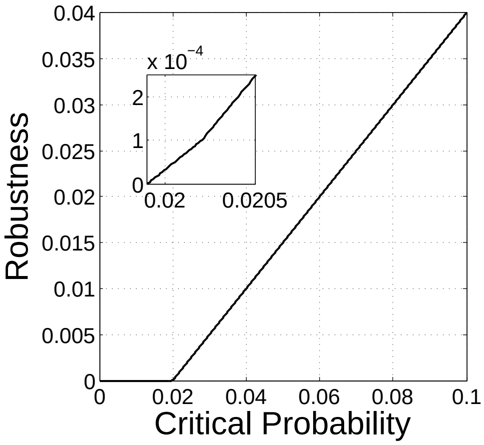

Figure 1: Serial robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

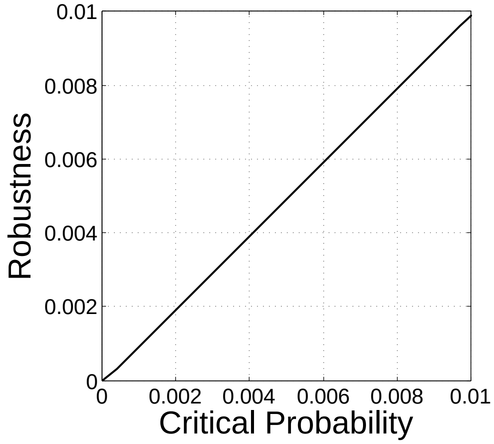

Figure 2: Parallel robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

§ Trade-off: Robustness vs. critical probability

§

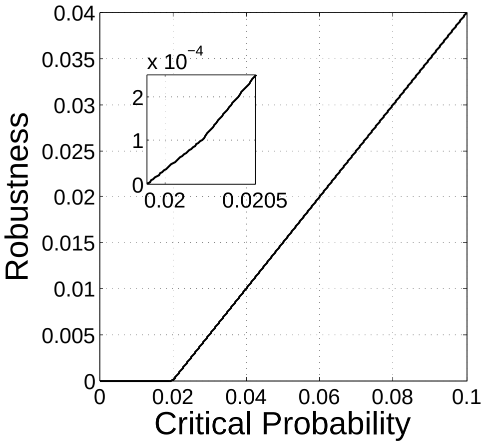

Figure 3: Serial robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

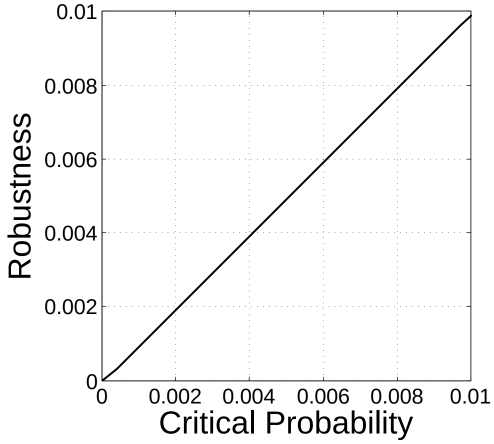

Figure 4: Parallel robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

§ **Trade-off: Robustness vs. critical probability**

§ **Zeroing: No robustness of estimated failure probability**

§

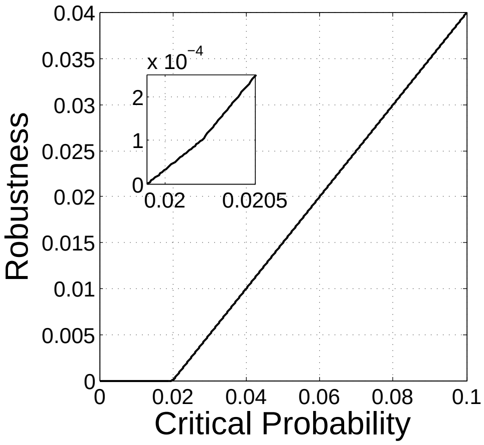

Figure 5: Serial robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

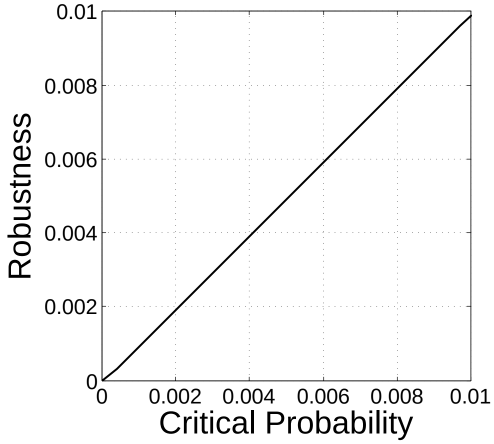

Figure 6: Parallel robustness. $\widetilde{F}_i = 0.01$, $\widetilde{F}_{12} = 0.0001$.

§ **Trade-off: Robustness vs. critical probability**

§ **Zeroing: No robustness of estimated failure probability**

§ **High cost of rbs: large slope.**

- **Serial:** $\Delta\widehat{h}/\Delta F_{\mathrm{cs}} = \mathbf{0.5}$.
- **Parallel:** $\Delta\widehat{h}/\Delta F_{\mathrm{cp}} = \mathbf{1}$.

§ Summary of

Reliability, Redundancy, Uncertain Correlation:

Analysis of uncertainty is essential.

1.3 *Parameter Uncertainty and* FAT *Tails*

§ **Basic idea:**

- **Fat tails:**
  - **Tail decays slower than exponential.**
  - **Not all moments exist.**
- **Thin-tail pdf may have uncertain parameters.**
- **Total pdf may be fat tailed.**

Example of Fat Tails

§ Exponential distribution of t.

- **t is a random variable, e.g. lifetime:**

$$f(t|\lambda) = \lambda \mathrm{e}^{-\lambda t}, \quad t \geq 0 \tag{12}$$

- **All moments of $t|\lambda$ exist.**

§

Example of Fat Tails

§ Exponential distribution of t.

- t is a random variable, e.g. lifetime:

$$f(t|\lambda) = \lambda e^{-\lambda t}, \quad t \geq 0 \tag{13}$$

- All moments of $t|\lambda$ exist.

§ Gamma distribution of λ

- λ is uncertain.

  E.g., mixture of populations.

$$\pi(\lambda) = \frac{\alpha}{\Gamma(k)}(\alpha\lambda)^{k-1}e^{-\alpha\lambda}, \quad \lambda \geq 0, \quad \underbrace{\alpha > 0, \quad k > 0}_{\textbf{parameters}} \tag{14}$$

- All moments of λ exist.

§ What is marginal distribution of t?

- $t|\lambda$ **is exponential (thin).**
- λ **is Gamma (thin).**
- t **is Pareto (fat).**
-

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- Define: $Y = 1 + \frac{T}{\alpha}$
- Y is Pareto: $f(y) = k\left(\frac{1}{y}\right)^{k+1}, \quad y \geq 1$
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§ What is marginal distribution of t?

- $t|\lambda$ **is exponential (thin).**
- λ **is Gamma (thin).**
- t **is Pareto (fat).**
- **Define:** $Y = 1 + \frac{T}{\alpha}$
- Y **is Pareto:** $f(y) = k \left(\frac{1}{y}\right)^{k+1}, \quad y \geq 1$
- **Mean and variance:**

$$\mathrm{E}(y) = \frac{k}{k-1} \quad \textbf{if} \quad k > 1$$

$$\mathrm{var}(y) = \frac{k}{k-2} - \left(\frac{k}{k-1}\right)^2 \quad \textbf{if} \quad k > 2$$

- **Not all moments exist.**
- **Parameter uncertainty implies fat tails.**

1.4 *Quantile Uncertainty*

§ Outcome, reward, r:

- **Random variable.**
- **Large is better than small.**

§

§ **Outcome, reward, r:**

- **Random variable.**
- **Large is better than small.**

§ **Decision: Choose system: mean, variance.**

§ **Problem: Distribution uncertain.**

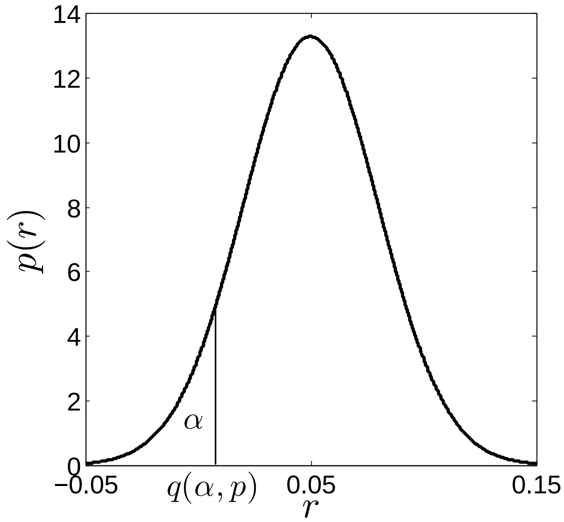

Figure 7: α quantile, $q(\alpha, p)$.

§ α Quantile, $q(\alpha, p)$:

$$\alpha = \int_{-\infty}^{q(\alpha,p)} p(r) \, \mathrm{d}r \tag{15}$$

- **α is probability of failure.**
- **$q(\alpha, p)$ is critical value of r.**

§ **Small quantile (far left) $\Longleftrightarrow$ high risk.**

- α **(e.g. $= 0.05$) is probability of loss $\geq |q(\alpha, p)|$.**
- **Suppose $q(\alpha, p)$ is very negative.**
- **Thus probability is α of very large loss.**

§

§ **Small quantile (far left) $\Longleftrightarrow$ high risk.**

- α **(e.g. = 0.05) is probability of loss** $\geq |q(\alpha, p)|$**.**
- **Suppose** $q(\alpha, p)$ **is very negative.**
- **Thus probability is** α **of very large loss.**

§ **Performance requirement:**

$$q(\alpha, p) \geq R_{\rm C} \qquad (16)$$

§

§ Small quantile (far left) $\Longleftrightarrow$ high risk.

- α (e.g. = 0.05) is probability of loss $\geq |q(\alpha, p)|$.
- Suppose $q(\alpha, p)$ is very negative.
- Thus probability is α of very large loss.

§ Performance requirement:

$$q(\alpha, p) \geq R_{\mathrm{C}} \tag{17}$$

§ Problem:

- pdf of r highly uncertain.
- May have fat tails.
- Hence $q(\alpha, p)$ highly uncertain.

§ Question: Is the system reliable?

§ Info-gap model of uncertainty:

- $\widetilde{p}(r)$ **is estimated pdf; e.g. normal.**
- **Envelope-bound uncertainty:**

$$|p(r) - \widetilde{p}(r)| \leq g(r)h \tag{18}$$

§

§ Info-gap model of uncertainty:

- $\widetilde{p}(r)$ **is estimated pdf; e.g. normal.**
- **Envelope-bound uncertainty:**

$$|p(r) - \widetilde{p}(r)| \leq g(r)h \tag{19}$$

§ How to choose envelope?

- **Engineering judgment.**
- **Dimensional analysis.**
- **Analogical reasoning.**
- **Assess "equivalent risk".**

§ Info-gap model of uncertainty:

- $\widetilde{p}(r)$ **is estimated pdf; e.g. normal.**
- **Envelope-bound uncertainty:**

$$|p(r) - \widetilde{p}(r)| \le g(r)h \tag{20}$$

§ Example: $g(r)$ is "$1/r^2$" on tails:

$$g(r) = \begin{cases} \dfrac{\widetilde{p}(\mu - r_{\rm s})(\mu - r_{\rm s})^2}{r^2} & \textbf{if } r < \mu - r_{\rm s} \\ \widetilde{p}(r) & \textbf{if } |r - \mu| \le r_{\rm s} \\ \dfrac{\widetilde{p}(\mu + r_{\rm s})(\mu + r_{\rm s})^2}{r^2} & \textbf{if } r > \mu + r_{\rm s} \end{cases} \tag{21}$$

Mean and variance may be unbounded.

§ Robustness:

Max horizon of uncertainty at which loss is acceptable:

$$\widehat{h}(\alpha, R_{\mathrm{C}}) = \max\left\{h : \left(\min_{p\in\mathcal{U}(h)} q(\alpha, p)\right) \geq R_{\mathrm{C}}\right\} \tag{22}$$

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Max horizon of uncertainty at which loss is acceptable:

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§ Robustness depends on:

- **Underlying design, e.g. μ, σ.**
- **Designated failure probability, α.**
- **Critical loss, R_C.**

§

§ Robustness:

Max horizon of uncertainty at which loss is acceptable:

$$\widehat{h}(\alpha, R_{\mathrm{C}}) = \max\left\{h: \left(\min_{p\in\mathcal{U}(h)} q(\alpha, p)\right) \geq R_{\mathrm{C}}\right\} \tag{24}$$

§ Robustness depends on:

- **Underlying design, e.g.** μ, σ.
- **Designated failure probability,** α.
- **Critical loss,** R_{C}.

§ Robustness is a decision function:

- **Satisfy requirements.**
- **Maximize robustness.**
-

§ Robustness:

Max horizon of uncertainty at which loss is acceptable:

$$\widehat{h}(\alpha, R_{\mathrm{C}}) = \max\left\{h : \ \left(\min_{p\in\mathcal{U}(h)} q(\alpha, p)\right) \geq R_{\mathrm{C}}\right\} \tag{25}$$

§ Robustness depends on:

- **Underlying design, e.g.** μ, σ.
- **Designated failure probability,** α.
- **Critical loss,** R_{C}.

§ Robustness is a decision function:

- **Satisfy requirements.**
- **Maximize robustness.**
- **Don't try to optimize the outcome (e.g.** $q(\alpha, p)$**).**

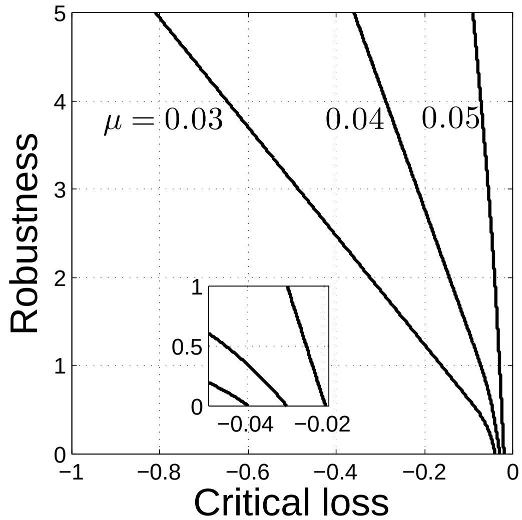

Figure 8: Robustness, $\widehat{h}$, vs. critical loss, R_{C}, for 3 values of μ. $\sigma = 0.03$.

§ Trade-off: robustness vs critical loss.

§

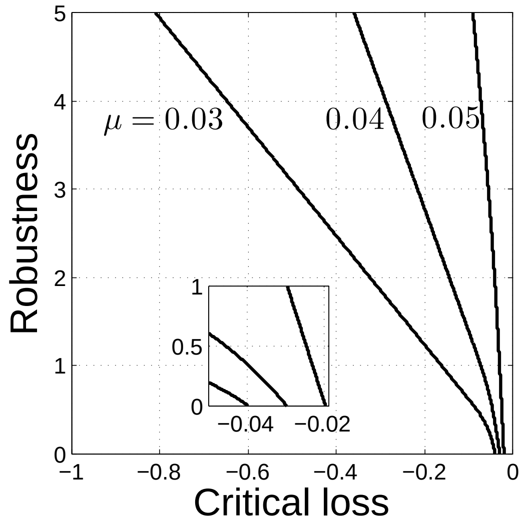

Figure 9: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for 3 values of μ. $\sigma = 0.03$.

§ **Trade-off: robustness vs critical loss.**

§ **Zeroing: No robustness of estimated critical loss.**

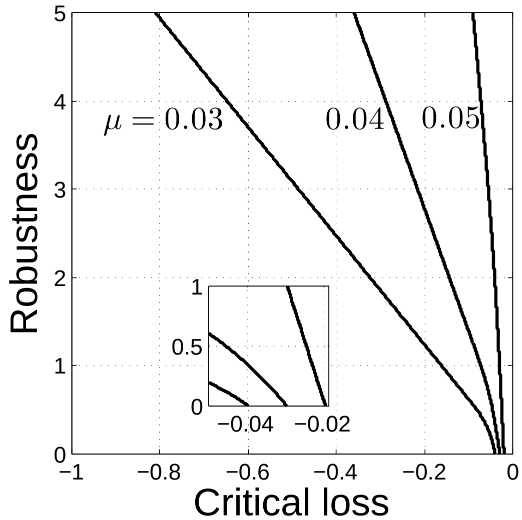

Figure 10: Robustness, $\widehat{h}$, vs. critical loss, R_{C}, for 3 values of μ. $\sigma = 0.03$.

§ Effects of increasing the mean:

- **Shift to higher robustness.**
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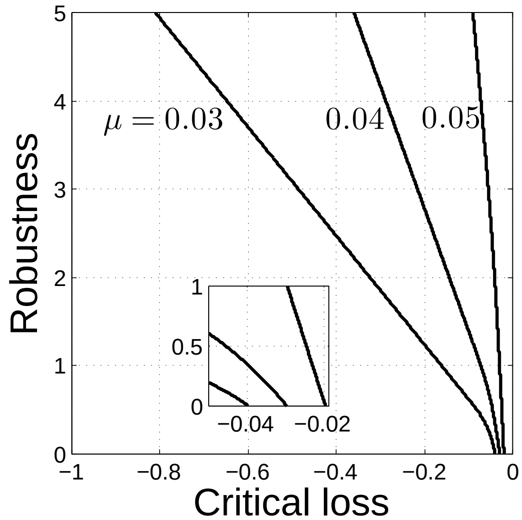

Figure 11: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for 3 values of μ. $\sigma = 0.03$.

§ Effects of increasing the mean:

- **Shift to higher robustness.**
- **Increase slope: reduce cost of rbs.**

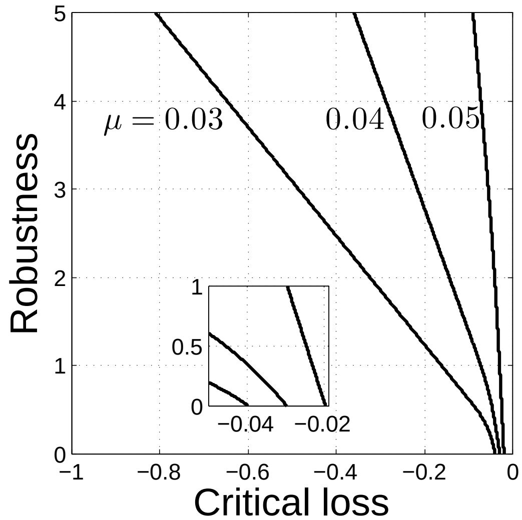

Figure 12: Robustness, $\widehat{h}$, vs. critical loss, R_{C}, for 3 values of μ. $\sigma = 0.03$.

§ Calibrating the robustness.

- **Is robustness of 3 or 5 "large"?**
- $\widehat{h} = 3$ **means 3** $\times$ **tail is tolerable.**
-

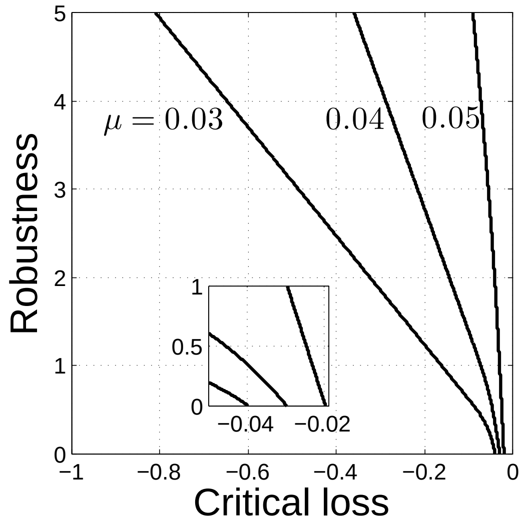

Figure 13: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for 3 values of μ. $\sigma = 0.03$.

§ Calibrating the robustness.

- **Is robustness of 3 or 5 "large"?**
- $\widehat{h} = 3$ **means 3** $\times$ **tail is tolerable.**
- **Standard normal below** -2σ**: 0.0228 (small probab.)**
-

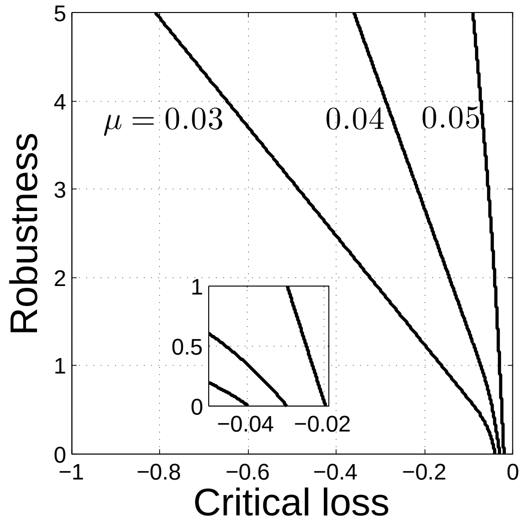

Figure 14: Robustness, $\widehat{h}$, vs. critical loss, R_{C}, for 3 values of μ. $\sigma = 0.03$.

§ Calibrating the robustness.

- **Is robustness of 3 or 5 "large"?**
- $\widehat{h} = 3$ **means 3** $\times$ **tail is tolerable.**
- **Standard normal below** -2σ**: 0.0228 (small probab.)**
- **Fat tail below** -2σ**: 0.1080. (not small probab.)**

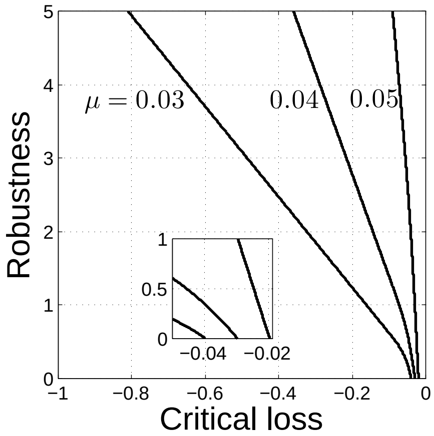

Figure 15: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for 3 values of μ. $\sigma = 0.03$.

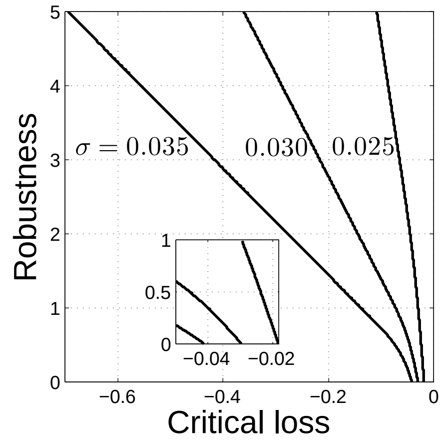

Figure 16: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for 3 values of σ. $\mu = 0.04$.

§ Mean-variance trade-off:

Increasing mean by 0.01

roughly equivalent to

decreasing st. dev by 0.005.

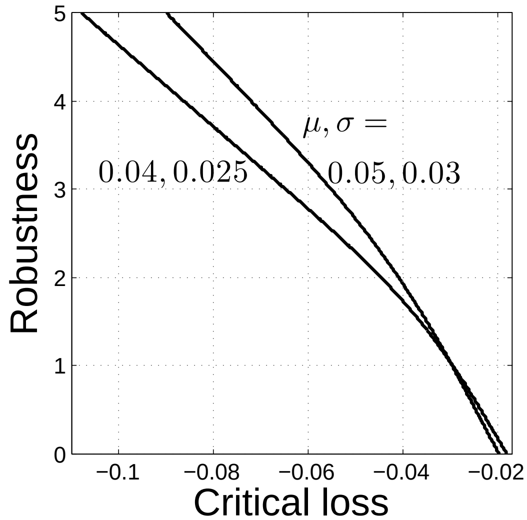

Figure 17: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for two different combinations of μ and σ.

§ Preference reversal:

- $(\mu, \sigma) = (0.04, 0.025) \succ_{\textbf{est.}} (0.05, 0.03)$.
-

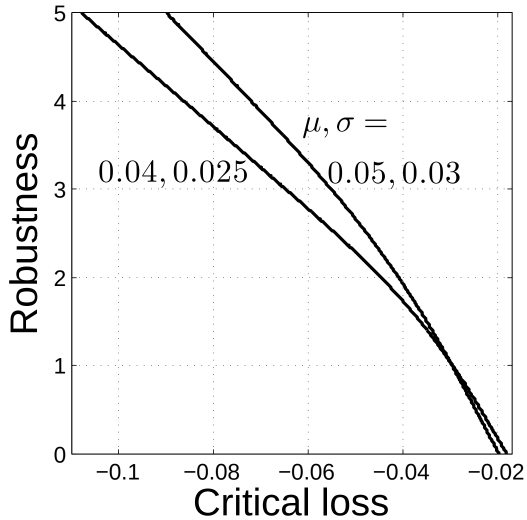

Figure 18: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for two different combinations of μ and σ.

§ Preference reversal:

- $(\mu, \sigma) = (0.04, 0.025) \succ_{\textbf{est.}} (0.05, 0.03)$.
- $(\mu, \sigma) = (0.05, 0.03) \succ_{\textbf{rbs.}} (0.04, 0.025)$ **at** $\widehat{h} > 1$.

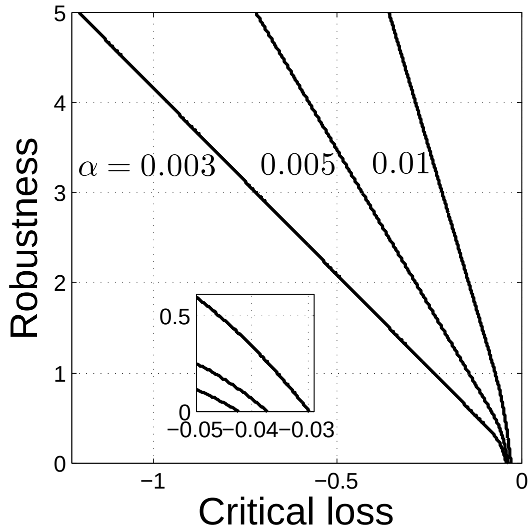

Figure 19: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for three different probabilities of failure, α. $\mu = 0.04$ and $\sigma = 0.03$.

§ Effect of greater probability of failure, α:

- **Greater robustness (curves shift right).**
-

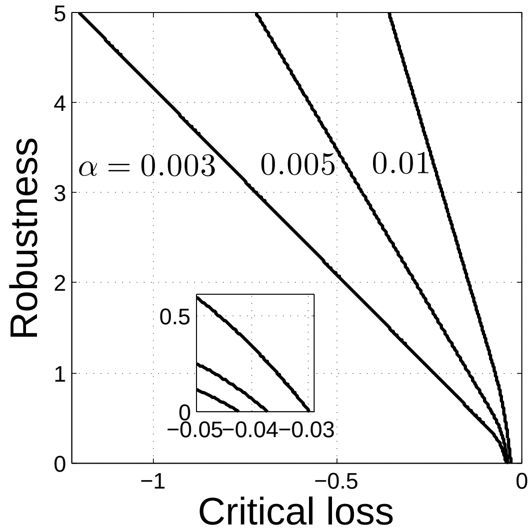

Figure 20: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for three different probabilities of failure, α. $\mu = 0.04$ and $\sigma = 0.03$.

§ **Effect of greater probability of failure, α:**

- **Greater robustness (curves shift right).**
- **Lower cost of robustness (steeper curves).**
-

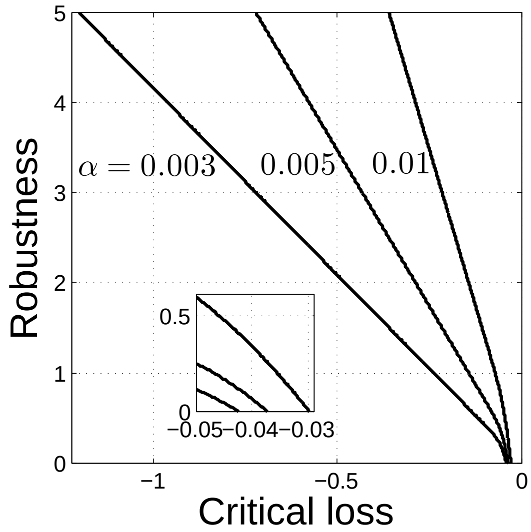

Figure 21: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for three different probabilities of failure, α. $\mu = 0.04$ and $\sigma = 0.03$.

§ Effect of greater probability of failure, α:

- **Greater robustness (curves shift right).**
- **Lower cost of robustness (steeper curves).**
- **Trade-offs: μ, σ and α.**

§ Less severe uncertainty:

- **Thin tails.**
- **Known pdf family.**
- **Uncertain moments of normal pdf:**

$$\mathcal{U}(h) = \left\{ p(r) \sim N(\mu, \sigma^2) : \ \left| \frac{\mu - \widetilde{\mu}}{\varepsilon_\mu} \right| \le h, \ \left| \frac{\sigma - \widetilde{\sigma}}{\varepsilon_\sigma} \right| \le h, \ \sigma \ge 0 \right\}$$

$$h \ge 0$$

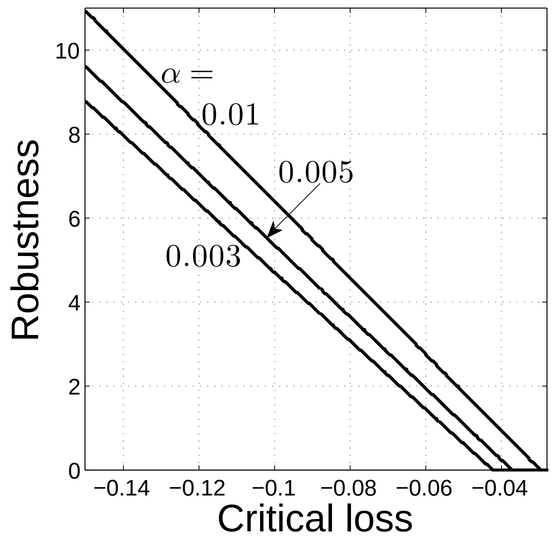

Figure 22: Robustness, $\widehat{h}$, vs. critical loss, $R_{\rm C}$, for three different probabilities of failure, α. $\widetilde{\mu} = 0.04$, $\widetilde{\sigma} = 0.03$, $\varepsilon_\mu = 0.004$, $\varepsilon_\sigma = 0.003$.

§ **Trade-off: Robustness vs. critical loss.**

§ **Zeroing: No robustness of estimated critical loss.**

§

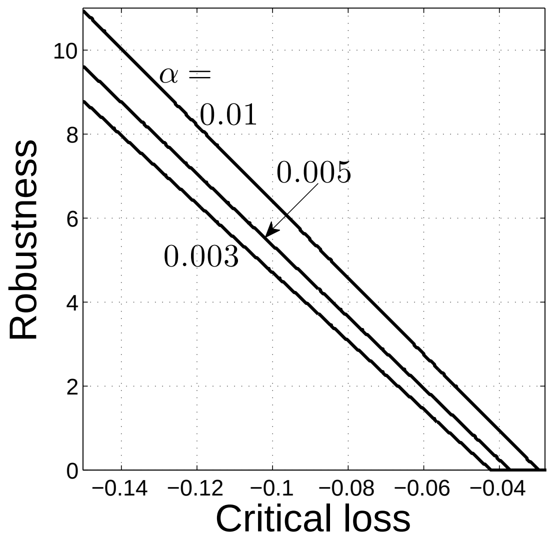

Figure 23: Robustness, $\widehat{h}$, vs. critical loss, R_{C}, for three different probabilities of failure, α. $\widetilde{\mu} = 0.04$, $\widetilde{\sigma} = 0.03$, $\varepsilon_\mu = 0.004$, $\varepsilon_\sigma = 0.003$.

§ **Trade-off: Robustness vs. critical loss.**

§ **Zeroing: No robustness of estimated critical loss.**

§ **Robustness: Greater than for fat tails.**

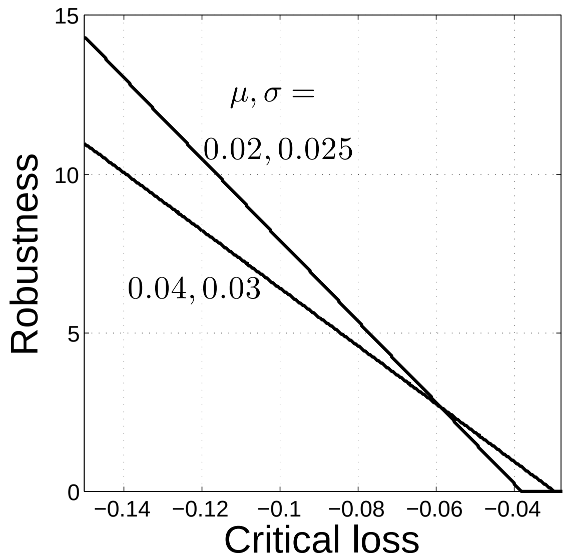

Figure 24: Robustness, $\widehat{h}$, vs. critical loss, R_C, for two different combinations of μ and σ. $\alpha = 0.01$.

§ Preference reversal:

- $(\mu, \sigma) = (0.04, 0.03) \succ_{\textbf{est.}} (0.02, 0.025)$.
- $(\mu, \sigma) = (0.02, 0.025) \succ_{\textbf{rbs.}} (0.04, 0.03)$ **at** $\widehat{h} > 2$.

§ Summary:

- **Uncertain pdf: fat tails.**
-

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§ **Summary:**

- **Uncertain pdf: fat tails.**
- **Nominal estimates: zero robustness.**
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  - **Robustness vs performance.**
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  - **Mean vs failure probability.**
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§ Summary:

- **Uncertain pdf: fat tails.**
- **Nominal estimates: zero robustness.**
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  - **Robustness vs performance.**
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  - **Mean vs failure probability.**
- **Cost of robustness.**
- **Preference reversal between options.**
- **Uncertainty judgment, info-gap model.**

Any Questions?