55. **Adaptive force balancing.** (p.173) A downward distributed load is applied on a straight unit interval. Denote the load $L(x)$ for $0 \le x \le 1$. Uncertainty in the load is described by:

$$\mathcal{U}(h) = \left\{ L(x) : \ \left| \frac{L(x) - \widetilde{L}}{\widetilde{L}} \right| \le h \right\}, \quad h \ge 0 \tag{183}$$

where $\widetilde{L}$ is known and positive. The designer must choose a distributed restoring force directed upward along the same unit interval. Denote the restoring force $R(x)$ for $0 \le x \le 1$. We require that the net moment of force around $x = 0$ not exceed the critical value M_{c}. Construct the robustness function for each of the following designs, and discuss your preferences among the designs:

(a) Designer 1 suggests choosing $R(x) = \widetilde{L}$.

(b) Designer 2 suggests an adaptive procedure whereby the restoring force is constant along the interval, and equal to the average of the actually realized force: $R(x) = \int_0^1 L(y)\,\mathrm{d}y$.

(c) Designer 3 suggests an adaptive procedure whereby the restoring force is constant along the interval, and equal to the average of the actually realized force: $R(x) = \int_0^1 L(y)\,\mathrm{d}y$. However, the adaptive procedure introduces additional uncertainty to the load, so eq.(183) is replaced by:

$$\mathcal{U}(h) = \left\{ L(x) : \ \left| \frac{L(x) - \widetilde{L}}{w\widetilde{L}} \right| \le h \right\}, \quad h \ge 0 \tag{184}$$

where $w > 1$ and known.

(d) Designer 4 suggests an adaptive procedure whereby the restoring force is linearly increasing along the interval, and equal at the midpoint to the average of the actually realized force: $R(x) = 2x \int_0^1 L(y)\,\mathrm{d}y$.

(e) Designer 5 suggests an adaptive procedure whereby the restoring force is linearly decreasing along the interval, and equal at the midpoint to the average of the actually realized force: $R(x) = 2(1 - x) \int_0^1 L(y)\,\mathrm{d}y$.