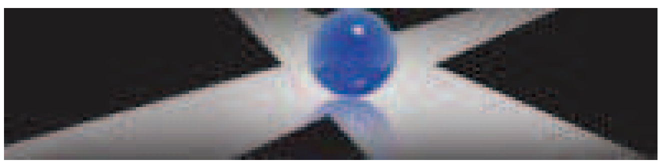
Lecture 2

Epidemiology with Uncertainty: Response to Bio-Terror

Yakov Ben-Haim

Technion

Israel Institute of Technology



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1 *Response to Bio-Terror*

§ **Analysis and planning:**

Use models and data to ameliorate impact of bio-terror attack.

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§ The challenge: Info-gaps.

- **Event scenario.**
- **Mass psychology.**
- **Epidemiological complexity.**
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§ Analysis and planning:

Use models and data to ameliorate impact of bio-terror attack.

§ The challenge: Info-gaps.

- **Event scenario.**
- **Mass psychology.**
- **Epidemiological complexity.**
- **Model error and incompleteness:**
  - **Structure.**
  - **Parameters.**
- **Data error.**
- **Numerical and analytical approximations.**

1.1 *Simplified Epidemiological Model*

§ Simplified epidemiological Model:

$$\frac{\mathrm{d}S(t)}{\mathrm{d}t} = -\gamma S(t)I(t)$$

$$\frac{\mathrm{d}I(t)}{\mathrm{d}t} = \gamma S(t)I(t) - \rho I(t)$$

$S(t) =$ **Number of susceptibles.**

$I(t) =$ **Number of infected.**

$\gamma =$ **constant infection rate.**

$\rho =$ **constant removal rate.**

Removal: death or recovery.

§ Approximate solution:

$$\begin{aligned} S(t) &= S_0 + g\mathrm{e}^{-\mu t} \\ I(t) &= I(0)\mathrm{e}^{-\rho t} + \frac{g\mu}{\rho - \mu}\left(\mathrm{e}^{-\mu t} - \mathrm{e}^{-\rho t}\right) \end{aligned}$$

$S_0 + g =$ **initial susceptible population.**

$S_0 =$ **final susceptible population.**

$\mu =$ **approximate infection rate.**

§ Total number of deaths:

$$\begin{aligned} N &= \int_0^\infty D\rho I(t)\,\mathrm{d}t \\ &\approx [I(0) + g]D \end{aligned}$$

$D =$ **fraction of "removed" who die.**

1.2 *Info-Gap Model of Uncertainty*

§ Uncertainties:

- **Analytical simplications.**
- **Modelling errors: Info-gaps.**

§

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§ Specifically:

$\widetilde{S}(t)$ = **known approx. susceptible pop.**

$S(t)$ = **unknown true susceptible pop.**

$$\left| \frac{S(t) - \widetilde{S}(t)}{\widetilde{S}(t)} \right| = \textbf{Unknown fractional error} \qquad (1)$$

§ Info-gap model, unknown fractional error:

$$\mathcal{U}(h, \widetilde{S}(t)) = \left\{ S(t) : \left| \frac{S(t) - \widetilde{S}(t)}{\widetilde{S}(t)} \right| \le h \right\}, \quad h \ge 0 \qquad (2)$$

- **Unknown $S(t)$ at uncertainty h.**
- **Unknown horizon of uncertainty h.**

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- **Unknown $S(t)$ at uncertainty h.**
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§ Info-gap model:

- **Unbounded uncertainty: $h \geq 0$.**
- **No worst case.**
- **Not min-max analysis.**

1.3 *Decisions and Their and Robustness*

§ Decisions:

- **Vaccinations: public and professionals.**
  - **Mass vaccinate.**
  - **Trace and vaccinate.**
- **Quarantine.**
- **Surveillance.**
- **Travel restrictions.**
- **Etc.**

§ Decisions influence:

- $\mu =$ **infection rate.**
- $\rho =$ **removal rate (death or recovery).**
- $g =$ **additional number of infected.**
- $I(0) =$ **initial number of infected.**
- $D =$ **fraction of "removed" who die.**
- **Etc.**

§ $q =$ decision vector: policy.

§ **Policy goal:**

Mortality $\leq N_{\rm c}$**.**

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§ **Mortality with policy** q**:**

- **Known estimated:** $N(q, \widetilde{S})$**.**
- **Unknown actual:** $N(q, S)$**.**

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§ **Policy goal:**

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§ Policy goal:

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- **Known estimated:** $N(q, \widetilde{S})$.
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§ Robustness of policy q:

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§ **Robust-satisficing strategy:**

- **Satisfice policy goal.**
- **maximize robustness.**

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§ Robustness of policy q:

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§ Robust-satisficing strategy:

- **Satisfice policy goal.**
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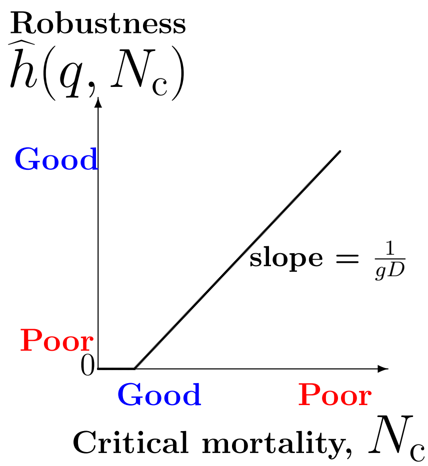
§ Robustness:

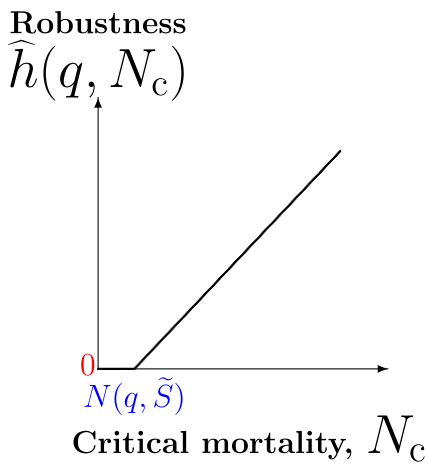
$$\widehat{h}(q, N_c) = \max \left\{ h : \left(\max_{S \in \mathcal{U}(h, \widetilde{S})} N(q, S) \right) \leq N_c \right\} \tag{4}$$

1.4 *Properties of the Robustness Function*



§ Trade-off: robustness up, performance down.

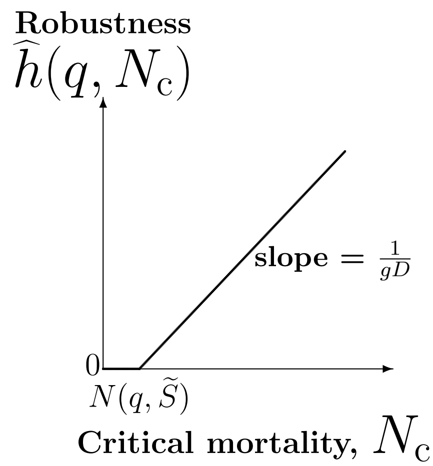
$$\widehat{h}(q, N_c) = \begin{cases} \frac{1}{gD}[N_c - N(q, \tilde{s})] & \textbf{if} \quad N(q, \tilde{s}) \le N_c \\ 0 & \textbf{else} \end{cases} \tag{5}$$



§ Zeroing:

no robustness of estimated performance:

$$\widehat{h}(q, N_c) = 0 \quad \textbf{if} \quad N_c = N(q, \widetilde{S}) \tag{6}$$



§ What does robustness mean?

- $\widehat{h}(q, N_c) = 0.1$: **robust to 10% error in $\widetilde{S}$.**
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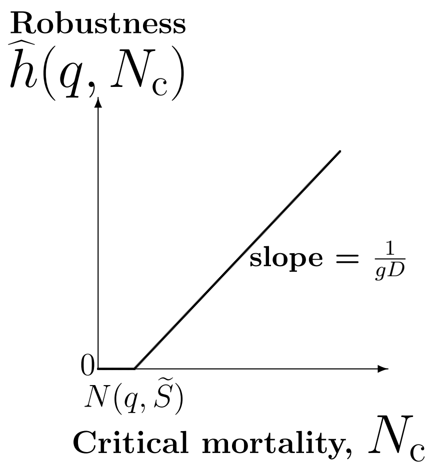

§ What does robustness mean?

- $\widehat{h}(q, N_c) = 0.1$: **robust to 10% error in** $\widetilde{S}$.
- **Slope** $= 1/gD \approx 1/(1000 \times 0.3) = 1/300$
- g **= # infected.** $D =$ **fraction of deaths.**
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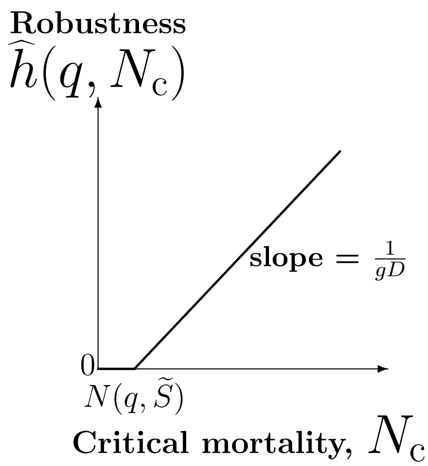

§ What does robustness mean?

- $\widehat{h}(q, N_c) = 0.1$**: robust to 10% error in** $\widetilde{S}$**.**
- **Slope** $= 1/gD \approx 1/(1000 \times 0.3) = 1/300$
- g **= # infected.** $D =$ **fraction of deaths.**
- N_c **up by 30:** $\widehat{h}(q, N_c)$ **up by 0.1**
- **50% robustness:** $N_c = N(q, \widetilde{S}) + 0.5 \times 300$**.**

1.5 *Preference Reversal and the Innovation Dilemma*

§ Two policies, q and $q^{\bullet}$:

- q: ρ, $I(0)$, g, D. **Innovative, hi-tech, less familiar.**
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- $q^{\bullet}$**:** $\rho^{\bullet}$, $I^{\bullet}(0)$, $g^{\bullet}$, $D^{\bullet}$. **State of the art, more familiar.**

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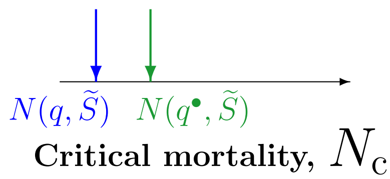
§ **Two policies, q and $q^\bullet$:**

- **q: ρ, $I(0)$, g, D. Innovative, hi-tech, less familiar.**
- **$q^\bullet$: $\rho^\bullet$, $I^\bullet(0)$, $g^\bullet$, $D^\bullet$. State of the art, more familiar.**

§ **Decision dilemma: which to choose?**

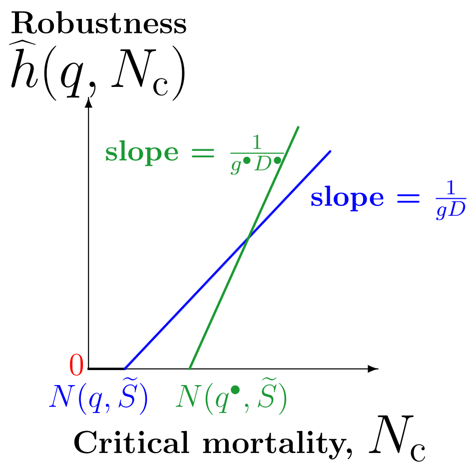


§ Nominal preference:

$$N(q, \widetilde{S}) < N(q^{\bullet}, \widetilde{S}) \tag{7}$$

implies nominal preference for innovative policy:

$$q \succ q^{\bullet} \tag{8}$$



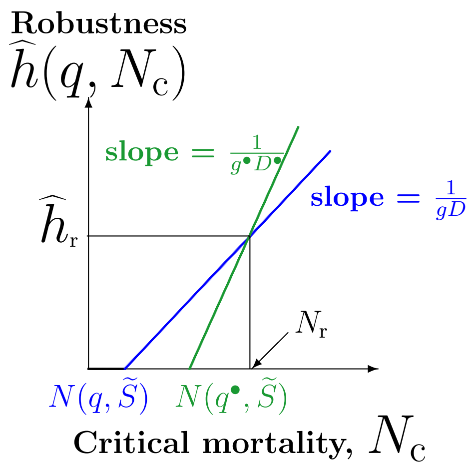
§ Nominal preference for innovative policy

$$q \succ q^\bullet \tag{9}$$

has zero robustness:

$$\widehat{h}(q, N_c) = 0 \quad \textbf{if} \quad N_c = N(q, \widetilde{S}) \tag{10}$$

and higher cost of robustness: lower slope.



§ Nominal preference: $q \succ q^\bullet$.

§ Preference reversal: innovation dilemma resolved.

- **If $N_c > N_r$ adequate, then $q^\bullet \succ q$.**
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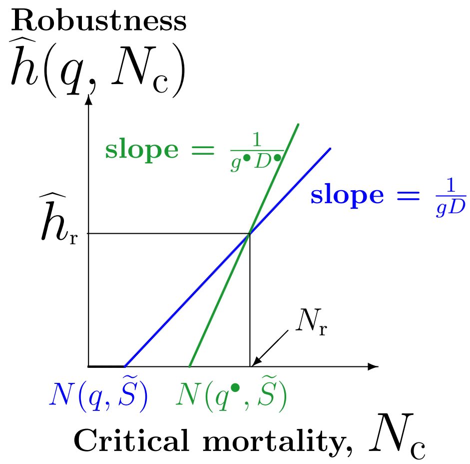
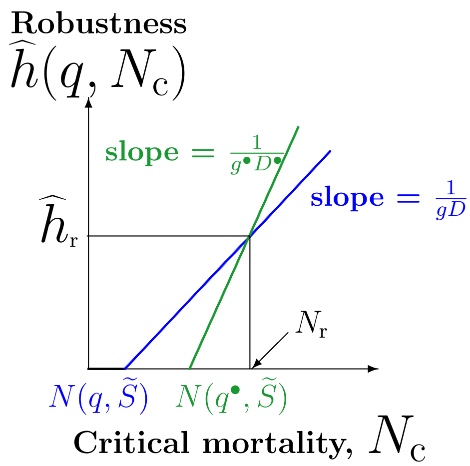


§ **Nominal preference:** $q \succ q^\bullet$.

§ **Preference reversal: innovation dilemma resolved.**

- **If** $N_c > N_r$ **adequate, then** $q^\bullet \succ q$.
- **If** $N_c < N_r$ **required, then** $q \succ q^\bullet$.

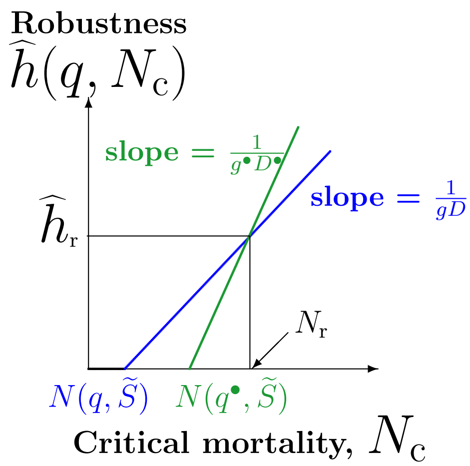


§ Example.

Parameter	Design q	Design $q^\bullet$
ρ	**0.05**	**0.06**
$I(0)$	**1000**	**1300**
g	**1000**	**800**
$N(q, \widetilde{S})$	**600**	**630**
$1/(gD)$	**0.00333**	**0.00417**

§ Nominal preference:

$$N(q, \widetilde{S}) = 600 < 630 = N(q^\bullet, \widetilde{S}) \quad \textbf{so} \quad q \succ q^\bullet \qquad (11)$$



§ Nominal preference:

$$N(q, \widetilde{S}) = 600 < 630 = N(q^\bullet, \widetilde{S}) \quad \textbf{so} \quad q \succ q^\bullet \tag{12}$$

§ Curves cross: slope$^\bullet$ > slope

If $\widehat{h}_r = 0.5$, $N_r = 750$ **adequate, then** $q^\bullet \succ q$

1.6 *Summary*

§ Use models to choose policy.

§

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§ **Models err: info-gaps.**

Hence: require robustness.

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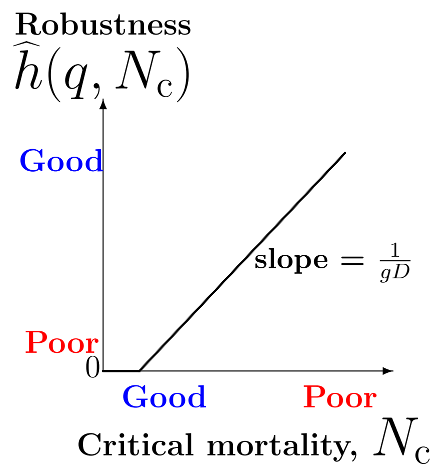
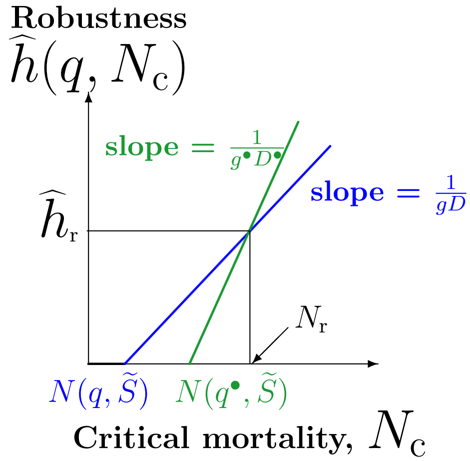
§ Nominal predictions: zero robustness.

§ Robustness trades-off against performance.



§ Robustness curves may cross:

Preference reversal: innovation dilemma resolved.



2 Conclusion

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innovation, discovery, ignorance, surprise.

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§ **Realism:** **our models are wrong now**
(and we don't know where or how much).

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In Conclusion

§ **Info-gap uncertainty:**
innovation, discovery, ignorance, surprise.

§ **Info-gap uncertainty is unbounded.**

§ **Optimism:** **our models get better all the time.**

§ **Realism:** **our models are wrong now**
(and we don't know where or how much).

§ **Responsible decision making:**

- **Specify your goals.**
- **Maximize your robustness to uncertainty.**
- **Study the trade offs.**
- **Exploit windfall opportunities.**