48. **Transmission in an uncertain medium** (p.224). We consider the transmission of a signal, or a disease, or a material object, in a medium that can absorb or amplify what is transmitted. However, the absorption or amplification by the medium is uncertain.

A "transmitter" is located a distance D from a "receiver". The absorptive property of the medium between transmitter and receiver is represented by a function $\rho(x)$. The function $\rho(x)$ can take both negative and positive values, where negative absorption means amplification. The strength at the receiver, of a signal of strength t at the transmitter, is:

$$r = t \exp\left(-\int_0^D \rho(x)\,\mathrm{d}x\right) \tag{152}$$

We require that the signal strength at the receiver be no less than a fraction f of the transmitted signal, where $0 < f < 1$ is specified by the design requirement.

(a) The absorption function is estimated to be constant at the value $\widetilde{\rho}$. The error of this estimate is approximated as σ, but the actual function can vary substantially and we have no reliable or meaningful worst-case estimate. An info-gap model for uncertainty in the absorption function is:

$$\mathcal{U}(h) = \left\{\rho(x) : \left|\frac{\rho(x) - \widetilde{\rho}}{\sigma}\right| \le h\right\}, \quad h \ge 0 \tag{153}$$

Derive an explicit expression for the robustness of the transmission. At what value of f does the robustness become zero? Explain the significance of this value. Explain the significance of the sign of the slope of the robustness curve for values of f at which the robustness is positive. Discuss the dependence of robustness on the distance between the transmitter and the receiver.

(b) The absorption function is estimated to be a sine function:

$$\widetilde{\rho}(x) = \sin\frac{2\pi x}{D_0} \tag{154}$$

where D_0 is a known constant. Negative absorption means amplification.

The error of this estimate is approximated as σ, but the actual function can vary substantially and we have no reliable or meaningful worst-case estimate. An info-gap model for uncertainty in the absorption function is:

$$\mathcal{U}(h) = \left\{\rho(x) : \left|\frac{\rho(x) - \widetilde{\rho}(x)}{\sigma}\right| \le h\right\}, \quad h \ge 0 \tag{155}$$

For any transmission distance D (which might differ from D_0), derive an explicit expression for the robustness of the transmission. Discuss the dependence of robustness on the distance between the transmitter and the receiver.