46. **Robustness and opportuneness of failure probability,** (p.194). The response of a system to input x is:

$$f(x) = \frac{a}{x} \tag{146}$$

where $a > 0$ and x is a random variable with an exponential distribution:

$$p(x) = \lambda \mathrm{e}^{-\lambda x}, \quad x \geq 0 \tag{147}$$

The failure criterion is probabilistic. The system fails if f exceeds f_{c}. The system requirement is that the probability of failure not exceed the critical value, P_{fc}.

(a) Derive an expression for the probability of failure, assuming that λ and a are known precisely.

(b) The coefficient a is estimated to equal $\widetilde{a}$ with error approximately s, and a is known to be positive. However, a may vary due to uncontrolled factors. Derive an expression for the robustness to uncertainty in a. What is the sign of the slope of the robustness curve? What does this sign indicate? At what value of critical failure probability does the robustness become zero?

(c) Continuing part 46b, consider the choice between two systems with parameters:

$$\lambda_1 < \lambda_2 \quad \text{and} \quad s_1 > s_2 \quad \text{and} \quad \lambda_1 s_1 < \lambda_2 s_2 \tag{148}$$

For what values of P_{fc} do you prefer option 1? Why? What do these three inequalities mean?

(d) Let P_{fw} be a lower probability than P_{fc}. Windfall occurs if the probability is no greater than P_{fw} that f exceeds f_{c}. Derive an expression for the opportuneness and discuss its relation to the robustness derived earlier. Specifically, at what value of $P_{\mathrm{fc}} = P_{\mathrm{fw}}$ do these curves cross one another, and what is the significance of this?