43. **Tichonov estimate with model uncertainty.** (p.189). We wish to choose the slope, s, of a linear scalar model:

$$y = sx \tag{133}$$

We have a prior estimate of the slope, $\widetilde{s}$, and we have data, (x_i, y_i), $i = 1, \ldots, M$. The Tichonov estimate of s minimizes:

$$T = \lambda(\widetilde{s} - s)^2 + (1 - \lambda)\frac{1}{M}\sum_{i=1}^{M}(y_i - sx_i)^2 \tag{134}$$

where $0 \le \lambda \le 1$. We will assume that x and y are dimensionless quantities.[4]

(a) Derive an expression for the estimate of s which minimizes T.

(b) Now consider model uncertainty, with two different info-gap models:

$$\mathcal{U}(h) = \{y = sx + u : \ |u| \le h\}, \quad h \ge 0 \tag{135}$$

$$\mathcal{U}(h) = \left\{y = sx + ux^2 : \ |u| \le h\right\}, \quad h \ge 0 \tag{136}$$

For each info-gap model, derive an expression for the robustness of an estimate of the slope. How does the robust-satisficing estimate differ between the two models? How do they differ from the Tichonov estimate? Note that, because x and y are dimensionless, the horizons of uncertainty in these two info-gap models are also dimensionless. This makes the robustnesses which are evaluated with these two info-gap models comparable.[5]

[4] If x and y have units, and even if they have the same units, then the units of T in eq.(134) are undefined. This means that the relative weights of the two terms in T are controlled by the units, not by the value of λ.

[5] If x and y have units then it is be necessary to calibrate the two robustnesses, which requires judgment and cannot be done uniquely. However, if x and y have units then we also face a different problem, noted in footnote 4.