Decision Making in an Uncertain World: Information-Gap Modeling in Water Resources Management

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***Abstract*—Information-gap (info-gap) modeling is put forth as a basic approach for enhancing decision making under uncertainty, especially when there is a high level of uncertainty and little information is available. The great need for having realistic techniques for describing severe uncertainty can be illustrated in water resources management by pointing out the wide range of uncertainties present in sustainable development when taking into account hydrological, socioeconomic, political, and other considerations. Some illustrative systems problems in watershed management are utilized to explain how info-gap modeling can be employed in practice.**

***Index Terms*—Hydrological cycle, information-gap models, sustainable development, systems modeling, uncertainty, water resources management.**

I. Introduction
The Pervasiveness of Uncertainty

THE mathematical concept of probability has become so familiar and widely adopted by society that it is often considered to be synonymous with the phenomenon of uncertainty. Consider, for example, a typical weather forecaster on the evening news who confidently declares that the chance of precipitation tomorrow is 80%. Since tomorrow has not yet taken place, there is certainly uncertainty as to whether or not it will rain. Furthermore, most people feel that a probabilistic statement regarding a future weather condition makes a lot of sense and is intuitively appealing.

Even though probability and statistics have almost become as familiar as common household items to most people, the fact is that there are a range of different situations in which people get uneasy feelings when probabilistic statements are uttered. For instance, when engineers and other proponents for building a large multipurpose dam in an earthquake zone calmly state that the risk of catastrophic dam failure anytime during the next 100 years is calculated to be a probability of 0.000 008 4 or 8.4×10^{-6}, residents living downstream from the proposed barrage feel very uncomfortable indeed. The inhabitants pose questions such as "What on earth does such a tiny number really mean?" "How can anyone possibly determine such a small quantity when a dam has never been built at the proposed location during the entire history of planet earth?" and "If the engineers are so clever, why can't they design a dam that is guaranteed never to fail?" In short, many citizens have no confidence in this kind of probabilistic assessment, which they believe to be highly unreliable and suspect.

What is required for more realistically understanding uncertainty in situations in which little information is available, and a probabilistic assessment appears to be like a "fish out of water," are alternative paradigms for describing different types of uncertainty. Accordingly, the major objective of this paper is to introduce the idea of information-gap (info-gap) modeling into the water resources and systems-engineering literature for formally structuring the uncertainty that exists between what we know about a system and what we would like to know. By being cognizant of the robustness-to-uncertainty of a solution for attempting to solve a given problem, especially in highly uncertain circumstances, society will be in a better position to make more informed and wiser choices. As explained in the paper, decision making under uncertainty is particularly important in water resources management, where physical, environmental, economical, social, political, and other uncertainties often must be taken into account. Similar approaches for better understanding uncertainty could be utilized for formally describing complex-systems problems arising in many different fields of engineering and elsewhere.

Motivating factors for having a variety of paradigms for systematically studying uncertainty in water resources are discussed in the next section. More specifically, in order to approach the ideal goal of sustainable development, one must be able to comprehend physical uncertainties within the hydrological cycle, uncertainties prevalent within society, and dependent interactions between society and the physical world in which it exists. Subsequent to defining info-gap models along with other associated concepts (including the robustness and opportunity functions), the formal mathematical modeling of real-world systems is discussed from a water resources management viewpoint. Finally, info-gap models are applied to some physical-systems problems arising in watershed management. A much more detailed presentation of Sections II and IV, as well as comparison of info-gap modeling to probability and fuzzy set theory, are available as a technical report [20].

II. Uncertainties in Water Resources Systems

A. Sustainable Development

The advancement of civilization and the accompanying dramatic increase in human population have brought about

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massive changes to the Earth's natural environment. Within the field of water resources, for example, the construction and operation of systems of multipurpose reservoirs have provided water supply and hydroelectric energy to support the growth of large cities and big industrial developments, as well as irrigation water for allowing more intensive agriculture and the expansion of agriculture into regions having little precipitation. The Chinese, for instance, currently are building what will become the largest single multipurpose reservoir in the world on the mighty Yangtze River (the well-known Three Gorges Project). Although large-scale water resources projects and other impressive undertakings by society have furnished many economic and social benefits, much of the progress has been made at the expense of our natural environment. More specifically, the ongoing devastation of our natural environment by civilization has caused adverse global changes, including global warming or climate change, stratospheric ozone depletion, decreased biodiversity, and pollution of our water, land, and air [30]. In response to these and other concerns, the World Commission on Environment and Development [37] proposed the concept of sustainable development, whereby the economic needs of society both now and in the future would be balanced against the necessity of maintaining a healthy environment [17], [31]. At the first Earth Summit, the United Nations Conference on Environment and Development, leaders from around the world met in Rio de Janiero in June 1992, to discuss a wide range of environmental and development issues fundamental to the establishment of a sustainable basis for life on earth. Even though agreements were reached on a range of important topics, in many cases, the countries of the world have failed to comply. For example, it appears that no nation will fulfill its commitment to reduce greenhouse gas emissions to 1990 levels by the year 2000 in order to help reduce global warming. As a matter of fact, in Canada and the United States, emission rates have continued to rise at a higher rate every year since 1992. Accordingly, at subsequent international negotiations on greenhouse gases held in Kyoto, Japan in early December 1997, a complex agreement was reached, in which developed nations would reduce greenhouse gases to specified levels in a little more than a decade, and one of the mechanisms for achieving this would be the introduction of tradeable emissions permits for the release of certain quantities of greenhouse gases.

In order to approach the ideal goal of sustainable development, we must first have a clear understanding of the natural system in which we live, the dynamics of the societal system in which we interact with one another, and the synergistic effects of these two interdependent systems. However, these dynamical systems and their interactions are extremely complex and thereby extraordinarily difficult to comprehend and formally model. This in turn means that we are confronted with a host of uncertainties that must be taken into account when attempting to make rational decisions that will permit a realistic balance between economic development and environmental integrity. Whether these uncertainties arise within a component of a given system, across a number of connected components, holistically at the overall systems level, or between dependent systems, the general structures of these uncertainties must be understood. In fact, we should be aware not only of how much we currently know about a given problem, but also we must be cognizant of the gap between that level of knowledge and what we need to know to fully comprehend the dimensions of the problem. The resulting uncertainty then can be modeled in the most appropriate fashion possible in order to enhance decision making within a sustainable-development framework. To appreciate the scope of uncertainties that are present in water resources-systems management and elsewhere, an overview is now presented, along with a brief discussion of the influence of society upon our natural environment.

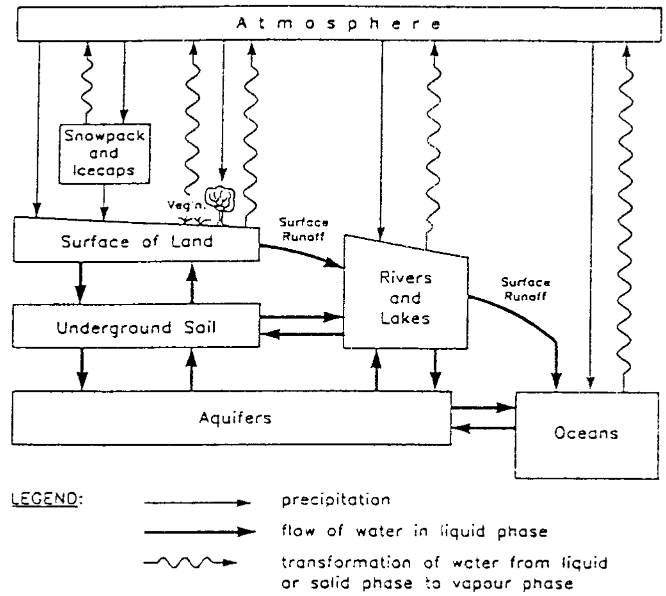

Fig. 1. Hydrological cycle.

B. Hydrological Cycle

Water is an essential ingredient for supporting life on Earth. Hence, when envisioning a physical system within which society can be sustained, an informative conceptual framework is the hydrological cycle. The basic idea underlying the hydrological cycle dates back more than 400 years to the time of the Renaissance, when Leonardo de Vinci proposed a systems model to portray the distribution and circulation of water on the surface of the land, underground, and in the atmosphere. As pointed out by authors such as White *et al.* [36] and Bennett and Chorley [6], there are many ways in which one can define environmental or natural systems within a hierarchy of systems models.

Fig. 1 displays a schematic of the hydrological cycle that is based upon the figure provided by Eagleson [10]. The throughput to the hydrological system in Fig. 1 is water that can occur in a liquid, solid, or vapor phase. Because the hydrological cycle does not allow water to escape, it forms a closed system with respect to water. Moreover, the fact that water is a physical entity with which everyone is familiar, makes water intuitively attractive as the system throughput, rather than a more abstract variable such as energy. The main agents that propel the water through the hydrological cycle are solar energy and gravity. As noted by Eagleson [10], the dynamic processes of vapor formation and transport are powered by solar energy, while precipitation formation and the flow of liquid water are driven by gravity. In fact, the transformation of water from one phase to another as well as the transportation of water from one physical location to another are the main features of the hydrological cycle.

To more rigorously understand what is taking place in the hydrological system displayed in Fig. 1, a wide variety of mathematical models have been developed. Some of the main types of mathematical tools that have been constructed for modeling different components in the hydrological cycle and some of their interconnections are referred to in Section IV-B. Whatever the case, the hydrological cycle possesses great complexity and is very difficult to model. For example, the flow of water through underground soil is hard to understand and model because of the heterogeneous composition of soils and bedrock. Moreover, pollution from industrial and agricultural activities that is spilled onto the land can pollute underground streams and aquifers. In fact, the hydrologic cycle furnishes a framework for understanding how human-made pollution can enter the hydrologic system at any point and pollute our entire environment by following the ancient pathways traced out by water in all of its forms. Therefore, the science of hydrology provides solid foundations upon which many other environmental sciences can build and interact. Whatever the case, high uncertainty exists in virtually all aspects of the hydrological cycle. There is a great need for realistic tools for capturing this uncertainty, so that reasonable decisions can be made with respect to human activities that take place within this system and influence its behavior. Authors such as Eagleson [10], Fallenmark [12], [13], and Kundzewicz [24] point out that human activity and decision making can have dramatic influences upon the hydrological cycle and hence are an integral part of that cycle. They also describe many key research areas in which the science of hydrology should be expanded so that sound environmental policies can be properly devised and implemented.

C. Interdependence of the Hydrological Cycle and Society

The hydrological cycle portrayed in Fig. 1 constitutes a systems model of key elements of the natural world upon which society is entirely dependent for its very existence. Civilization has cast an artificial web of international and other political boundaries across the entire surface of the land, rivers, and lakes shown in the bottom left portion of Fig. 1. Societies have expanded agricultural land and built huge cities on the surface of the land, extracted minerals using large open-pit mines and underground tunnels, utilized huge quantities of freshwater from rivers and lakes as well as underground aquifers, and exploited fish and other resources from the oceans. Previously, humankind considered water and other resources to be an almost infinite source of wealth and a huge sink in which to discharge unwanted byproducts. However, since the industrial revolution, and more dramatically during the past few decades, the activities of society have actually started to detrimentally affect various components of the hydrological cycle as well as other natural systems.

Larger river basins often encompass local political regions within a nation as well as territories in other countries. The Danube River Basin in central Europe, for instance, covers an expanse of 817 000 km^2 within 17 nations. Other examples of transboundary river basins include the Mekong, Zambezi, Great Lakes/St. Lawrence, and Rhine basins. Consequently, from the perspective of water resources management, a practical and realistic way to implement sustainable development and other related policies is at the level of river basins. Recently, the nations of the Danube formulated a strategic plan for cooperative sustainable development in the region for the time period from 1995 to 2005 [11]. Although this is only a meager beginning, the leadership shown by the Danube countries and elsewhere in the world is pointing societies in the correct direction.

III. Set Models of Info-Gap Uncertainty

Classically, uncertainty is represented by probability theory, which quantifies lack of information either in terms of the frequency of recurrence of events, or in terms of an observer's subjective degrees of belief [27]. In recent decades, the theory of fuzzy logic has emerged to provide a range of alternatives to probability theory. Various types of uncertainties are quantified with fuzzy membership functions, such as the linguistic ambiguity of a proposition, or the possibility or the necessity (rather than the probability) of occurrence of an event [1], [9], [22], [38], [39].

In the historical evolution of uncertainty-thinking, fuzzy logic is a major break from traditional probability theory. Each of these theories reflects a different aspect of imperfect or fragmentary information, and each does so in a different way. Furthermore, each has developed from different classes of applications. But despite all their differences, they share a similarity of form. Both fuzzy logic and probability quantify uncertainties with normalized mathematical functions: the probability density or the membership function.

Can we quantify uncertainty without using distribution functions at all? The answer, of course, is yes, and the need to do so arises when information is scarce. Specifically, the need for a sparse model of uncertainty occurs when we find ourselves far over the frontier of our firm knowledge and deep in the realm of the unknown. In such situations, we must deal with information that is much more deficient than is customarily handled by either probability or fuzzy logic.

Consider the following situation. Toxic waste is released from an industrial plant into a river and flows downstream causing damage to plant and animal life, as well as reducing the quality of the water for human use. In assessing the environmental impact downstream, it is necessary to account for the local indigenous rate of absorption, consisting of adsorption and chemical reaction of the toxic materials with the riverbed and with other materials dissolved in the river-water. These processes are complex and depend on many factors such as flow rate, degree of turbulence, temperature, concentration, and so on. These are poorly known and vary in time and place along the river. In short, the rate of toxic waste removal from the river depends on a function of time, space, and other factors. Hydrological and water-quality models can be developed to describe these removal processes, but enormous resources are required to verify the reliability of a comprehensive model. In lieu of the extensive resources of money and time needed to develop a well-grounded realistic model, an approximate model can be adopted with the realization that it will deviate by an unknown but possibly substantial margin from the actual behavior. In employing an approximate model, we are acknowledging a large information gap between what *is known* and what *needs to be known* in order to make rational decisions in the management of the toxic effluent.

This information gap is a severe form of uncertainty and occurs often in practice. In many innovative technological projects, very little that should be known in order to make wise and reliable decisions is actually known. In environmental planning, in industrial management, in social and economic decision making, in medical diagnosis, and in other areas, quite often the gap between what we know and what we need to know is substantial. How can we organize our information, and the lack thereof? How can we measure the size of this gap to get a meaningful and robust quantification of the uncertainty confronting the decision makers?

Returning to the toxic-effluent example, let $\overline{r}(x,t,q)$ denote the removal rate as a function of position x, time t, and other quantities q, based on the best available (but possibly quite simple) model. The actual removal-rate function $r(x,t,q)$ deviates in an unknown manner from the nominal model $\overline{r}(x,t,q)$. We have no information with which to express the likelihood of various alternative rate functions, so we are unable to specify a probabilistic model for the uncertainty in the function $r(x,t,q)$. In fact, since the phenomenon in question is quite complex, and since $r(x,t,q)$ itself is a multi-argument function and not simply a fixed number or vector, we are hard-pressed to make any useful assertions about plausibility among the infinite continuum of alternative r-functions. For this reason, a fuzzy model of the uncertainty in the removal-rate function is inaccessible without introducing far-reaching and unverified assumptions. In light of this severe lack of information about the possible variations of the rate function, a simple set model of uncertainty could be formulated as the set of all functions consistent with the nominal function $\overline{r}(x,t,q)$ up to a given level of deviation

$$\mathcal{U}(\alpha,\overline{r}) = \{r(x,t,q)\colon |r(x,t,q) - \overline{r}(x,t,q)| \leq \alpha\}, \quad \alpha \geq 0. \tag{1}$$

$\mathcal{U}(\alpha,\overline{r})$ is the set of all functions whose deviation from $\overline{r}(x,t,q)$ is nowhere greater than α. For a fixed value of α, this set represents uncertainty in the rate function by specifying a range of variation of $r(x,t,q)$ around the nominal rate function $\overline{r}(x,t,q)$. The larger the value of α, the greater the range of unknown variation, so α is called the uncertainty parameter. However, quite often the value of α itself is not known, so in fact (1) is not a single set but rather a family of nested sets. The degree of nesting, as well as the level of uncertainty, is expressed by the uncertainty parameter α. The family of nested sets $\mathcal{U}(\alpha,\overline{r})$ $\alpha \geq 0$ is an *info-gap model of uncertainty*. Since the sets $\mathcal{U}(\alpha,\overline{r})$ in this family are in fact convex sets, this is a convex info-gap of uncertainty (also called a convex model). The info-gap model in (1) is only one of a wide range of commonly used info-gap models. The formulation and choice of an info-gap model depends on the type of initial information. The theoretical development of info-gap models with applications to nuclear assay [2], mechanical analysis [5], and reliability theory [3] are presented elsewhere.

When we organize our information (and our ignorance) in terms of families of sets or clusters like this, the decision maker faces sets of rate functions that may be confronted. Which function will actually be confronted is unknown. Probability and possibility theories also have us think about sets of events. In probability theory, we might evaluate the frequency of recurrence of rate functions whose value is less than ρ, for instance. In possibility theory, we might ask whether the set of functions with low rates is highly possible. The difference is that with info-gap models of uncertainty, we organize the events into clusters, but we do not employ distribution functions to measure them. We simply do not have sufficient information to formulate a probability density or a membership function.

The distribution functions in probability or fuzzy theory are designed to measure uncertainty and are related in particular ways to the size of sets. Large sets will tend to have a large probability of including events that frequently recur or that quite possibly will happen. But here again is the crux of the difference between distribution-based uncertainty models and info-gap set-models. In info-gap models of uncertainty, we can rank degrees of the information gap in terms of the size of the uncertainty parameter α, but this is much weaker information than probability or possibility, in which the distribution functions indicate recurrence frequency or plausibility. In info-gap set models of uncertainty, we concentrate on cluster-thinking rather than on recurrence or likelihood. Given a particular quantum of information, we ask: "What is the cloud of possibilities consistent with this information?" "How does this cloud shrink, expand, and shift as our information changes?" "What is the gap between what is known and what could be known?" We have no recurrence information, and we can make no heuristic or lexical judgments of likelihood.

Info-gap modeling is a stark theory of uncertainty, motivated by severe lack of information. It does, however, have its own particular subtlety. It is facile enough to express the idea that uncertainty may be either pernicious or propitious. That is, uncertain variations may be either adverse or favorable. Adversity is the possibility of failure, while favorability is the possibility of sweeping success. The *robustness function* is the greatest level of uncertainty consistent with no-failure. The *opportunity function* is the least level of uncertainty that entails the possibility of sweeping success. If q is a vector of parameters such as time, design variables, and model parameters, we can succinctly express the robustness and opportunity functions as the maximum or minimum of a set of α-values

$$\hat{\alpha}(q) = \max\{\alpha\text{: minimal requirements are satisfied}\} \quad \text{(robustness)}. \tag{2}$$

$$\hat{\beta}(q) = \min\{\alpha\text{: sweeping success is obtained}\} \quad \text{(opportunity)}. \tag{3}$$

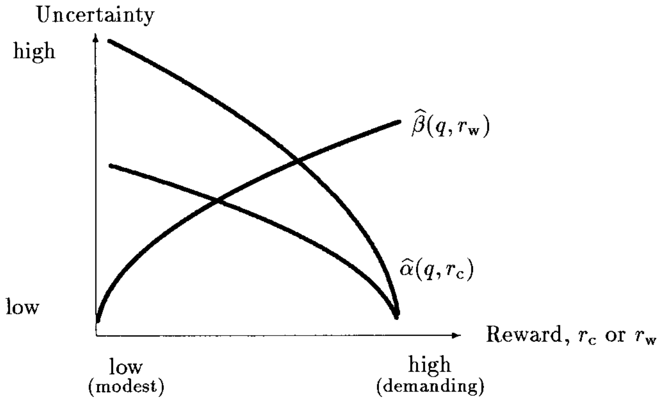

Fig. 2. Two robustness curves and one opportunity curve (schematic).

The robustness function $\hat{\alpha}(q)$ is the immunity against failure, so a large value of $\hat{\alpha}(q)$ is desirable. In contrast, the opportunity function $\hat{\beta}(q)$ is the immunity against sweeping success, so a small value of $\hat{\beta}(q)$ is desirable.

Quite often, the degree of success is assessed by a scalar reward function $R(q,u)$, which depends on the vector q of actions, decisions, and model parameters as well as on an uncertain quantity u whose variations are described by an info-gap $\mathcal{U}(\alpha, \overline{u})$. The minimal requirement in (2) is that the reward be no less than a critical value r_c. Likewise, the sweeping success in (3) is attainment of the "wildest dream" level of reward r_w. The robustness and opportunity functions can now be expressed more explicitly

$$\hat{\alpha}(q, r_c) = \max\left\{\alpha\colon \min_{u \in \mathcal{U}(\alpha, \overline{u})} R(q,u) \geq r_c\right\} \tag{4}$$

$$\hat{\beta}(q, r_w) = \min\left\{\alpha\colon \max_{u \in \mathcal{U}(\alpha, \overline{u})} R(q,u) \geq r_w\right\} \tag{5}$$

As explained elsewhere [4] and illustrated in Fig. 2, the robustness function $\hat{\alpha}(q, r_c)$ decreases monotonically in the minimal required reward r_c. This expresses the tradeoff between demanded reward and immunity to uncertainty. If a large reward is required for survival, then only low immunity to uncertainty is possible. Conversely, the opportunity function $\hat{\beta}(q, r_w)$ increases monotonically in wildest-dream reward r_w. Sweeping success cannot be attained at low levels of ambient uncertainty.

The locations of the robustness and opportunity curves on the uncertainty-versus-reward plane, as in Fig. 2, reveal the type of gambling that is expressed by these tradeoffs. Consider the upper robustness curve $\hat{\alpha}(q, r_c)$, which falls to low and vulnerable levels of uncertainty only at high-demanded reward. Different models and prior information lead to the lower robustness curve which, though still decreasing monotonically with r_c, runs more closely to the origin. The upper robustness curve represents bolder behavior than the lower curve. At any given level of demanded reward, a greater level of ambient uncertainty is tolerable according to the upper curve. Conversely, at fixed ambient uncertainty, the upper robustness curve allows greater demanded reward than the lower curve. Ascribing these two robustness curves to two different decision makers (or to two distinct decision strategies), we can say that the lower decision maker is more sensitive or averse to uncertainty than the upper decision maker. Equivalently, the upper curve will lead the decision maker to behavior that would look risky or rash when viewed through the strategy of the lower robustness curve.

IV. Mathematical Modeling of Systems

A. Deterministic and Indeterminate Models

Human beings develop mathematical models to capture the key characteristics of a system or components of a system in order to have a better understanding of how the process works. This enables them to make improved decisions about what to do to solve a given problem. For example, when deciding upon how to ameliorate the flow of pollution through the soil caused by previous dumping of liquid chemicals on the land by local chemical plants, finite elements can be utilized for mathematically modeling the physics of the problem and tracing where the pollutants may travel [15], [32], [35]. When designing a large-scale engineering project such as a regional irrigation project, the graph model for conflict resolution [14] can be employed for determining compromise resolutions to the dispute arising among the proponents of the project, environmental groups, government agencies, and other interest groups. By definition, any mathematical model is an approximation of reality, since it can never be the actual physical or social system that is being modeled in the real world. Nonetheless, by developing a model or set of models, which is as simple as possible according to the principle of Occam's Razor (yet provides a reasonable explanation of what is happening), human beings have at their disposal a common communication medium for discussing and better comprehending the problem at hand when trying to decide what to do in a responsible, fair, and equitable manner.

One can classify mathematical models as being deterministic or indeterminate. A deterministic model is one in which specified conditions completely establish future consequences. A deterministic model contains no uncertainty components based on the probability, fuzzy sets, or info-gap modeling described in the previous section. An example of a purely deterministic model is a set of differential equations developed for describing a physical system. With an indeterminate model, on the other hand, antecedent conditions do not fix unique consequences. An indeterminate model contains probabilistic, fuzzy, info-gap or some other version of uncertainty, possibly together with a deterministic model. Examples of indeterminate models of water resources systems containing info-gap and probabilistic uncertainty are discussed in Section V.

One could argue, no doubt, that a carefully constructed model furnishes a formal description of a physical or social system, thereby removing substantial uncertainty. However, some uncertainty remains even if the model is quite sophisticated. There are many reasons for the persistence and pervasiveness of uncertainty. All models are approximations of reality which, whether natural or societal, is extremely complex and dynamically changing over time. Inaccuracies and uncertainties may enter in modeling the linkage between fundamentally different types of systems, such as in combining

TABLE I
CLASSIFICATIONS OF STOCHASTIC MODELS

		STATE SPACE	
		Discrete	**Continuous**
TIME	**Discrete**	Markov Chain	Time Series Models
	Continuous	Point Processes	Stochastic Differential Equations

TABLE II
CLASSIFICATIONS OF DECISION-MAKING TECHNIQUES

		OBJECTIVES	
		One	**Two or More**
DECISION	**One**	Most OR Methods	Multiple Criteria Decision Making (MCDM)
MAKERS	**Two or More**	Team Theory	The Graph Model for Conflict Resolution

physical and socioeconomic models. Data may be deficient and measurements may be imprecise. Furthermore, some phenomena may themselves be inherently uncertain. Nonetheless, models provide a valuable focal point for discussion and a springboard for enhanced understanding and wise decision making.

For a long time, mathematical modeling has been intensively employed in water resources for tackling a wide variety of challenging problems. The objective of the next two sections is to outline some of the main types of mathematical models that have been developed and employed in the field of water resources for describing physical-systems and societal-systems problems. Overall, probability has been utilized extensively in both physical and societal-systems modeling. However, fuzzy sets have only been utilized in a few instances such as in multiple-criteria decision making. Info-gap modeling has never been used in water resources and some ways in which it could be utilized are illustrated in Section V. By having an array of useful approaches for modeling uncertainty, the authors believe that decision makers will be in a more informed position for deciding how to properly balance economic development and maintain a healthy natural environment.

B. Physical-Systems Models

Integral and differential calculus have been extensively utilized to model most aspects of the hydrological cycle shown in Fig. 1. For example, sets of differential equations have been used for modeling both surface and groundwater problems. Because a system of differential equations is often extremely difficult to solve analytically, difference equations and finite elements are commonly used in practical applications. One of a number of main challenges is to link general circulation models largely developed in the field of meteorology, with more localized hydrological models. Geographic information systems provide a valuable means for systematically storing information and furnishing valuable data for calibrating large-scale models.

Stochastic models allow for the order of occurrence of probabilistic events to be taken into account. Following Cox and Miller [8], Table I describes a method for categorizing stochastic models according to the two criteria of time and state space. Notice that time can be either discrete or continuous and the state space or values of the variables describing the system being modeled also can be subdivided according to discrete and continuous values. Illustrations of four kinds of models that can be categorized using the above criteria are given in Table I. Markov chains, for instance, fall under the subdivision of stochastic models, which incorporate discrete time and discrete values of the state space in their mathematical structure. Stochastic differential equations can handle continuous time and continuous values of the state space [21], [23]. Point processes, such as Poisson processes, model discrete values over continuous time. Stochastic models falling in all categories in Table I have been employed for addressing problems arising in water resources [19], [21]. For example, when deciding upon the most economic design of a multipurpose reservoir, a time-series model fitted to the historical river flows can be used for simulating other possible flow sequences that can be employed for testing alternative reservoir designs and isolating the optimal design. Singh [32] and Ward and Elliot [35] provide comprehensive handbooks on the employment of mathematical models in environmental hydrology. Moreover, Hipel and McLeod [21] provide an informative list of key references in stochastic processes, time series analysis, statistics, stochastic water-quality modeling, stochastic hydrology, data collection, and forecasting.

C. Societal-Systems Models

By societal-systems models, we are referring to formal models in decision making that take into account economic, social, or political considerations as well as pertinent results arising from physical-systems modeling. As explained in the water-resources literature by Hipel [18] and other authors, since the end of World War II, more decision-making techniques have been developed in operational research (OR) than in any other field. OR consists of some general methodologies and many specific techniques for studying decision-making problems. The British initiated OR just prior to World War II when they performed research studies into the operational aspects of radar systems for detecting incoming enemy aircraft to the United Kingdom. Throughout the war, the British employed OR in all of their military services for successfully solving large-scale military problems involving the movement of great numbers of military personnel and huge quantities of war materials [7], [34]. The American military also used this systems-science approach to problem solving during the second world war, but called it operations research. Since the war, OR has been extensively expanded and utilized for looking at operational problems in many different fields outside of the military, such as management sciences, transportation engineering, water resources, and industrial engineering. OR societies have sprung up in most industrialized countries, along with the publication of many OR journals.

Table II shows how OR methods can be categorized according to the criteria of number of decision makers and number of objectives. As shown in that table, most OR techniques reflect the viewpoint of one decision maker hav-

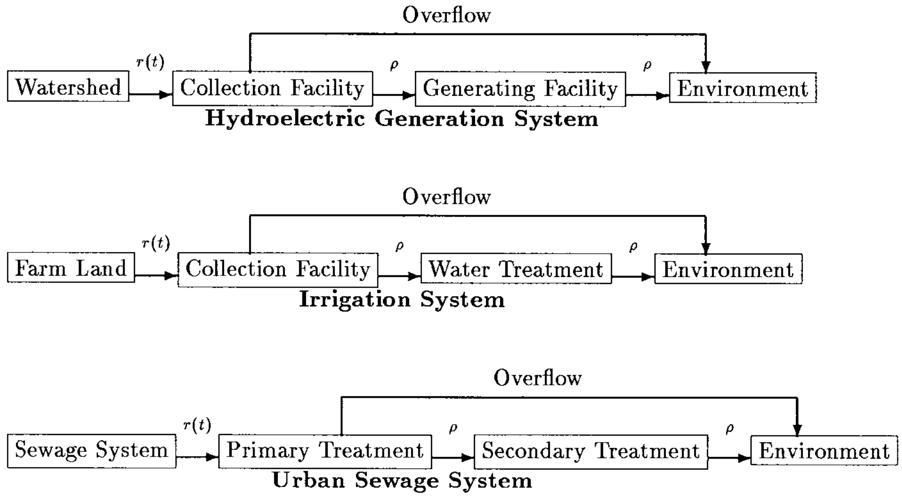

Fig. 3. Three interpretations of the hydrological model studied in the examples.

ing one objective. Optimization techniques, including linear and nonlinear programming, fall under this category because usually they are employed for minimizing costs in terms of dollars or maximizing monetary benefits from one group's viewpoint, subject to various constraints. Often, both economical and physical constraints can be incorporated into the constraint equations in optimization problems. Many of the probabilistic techniques like queueing theory, inventory theory, decision theory, and Markov chains fall under the top left cell in Table II. An example of a technique designed for handling multiple objectives for a single decision maker is multiple criteria decision making (MCDM). This method is designed for finding the more preferred alternative solutions to a problem in which discrete alternatives are evaluated against criteria ranging from cost (a quantitative criterion) to aesthetics (a qualitative criterion). The evaluations of the criteria for each alternative reflect the objectives or preferences of the single decision maker. MCDM constitutes one set of decision tools in which fuzzy uncertainty has been utilized [16], [29], [33]. In Table II, team theory is categorized according to multiple decision makers and one objective, because in a sporting event for instance, each team has the single objective of winning. Finally, the graph model for conflict resolution [14] constitutes an example of a technique in Table II that can be used for modeling and analyzing disputes in which there are two or more decision makers, each of whom can have multiple objectives. Aside from methods from the field of OR, approaches for modeling conflict resolution and other types of decision situations have been developed in the areas of cognitive science and psychology (see, for instance, Neale and Bazerman [28] and references given therein). Whatever the case, since the early 1960's, OR techniques have been applied extensively to water resources management problems (see [18], [21], [25], and [26], and references contained therein). Currently, the authors are developing an info-gap component for the graph model for conflict resolution in order to represent the uncertainty present in the preferences of decision makers. However, in the next section, info-gap models are incorporated into physical-systems models of hydrological problems.

V. WATERSHED MANAGEMENT WITH INFO-GAP UNCERTAINTY: SOME EXAMPLES

A. Hydrological Model

Consider a very simple hydrological model, for which three scenarios are shown in Fig. 3. In the hydroelectric generation system, water flows from the watershed to a collection facility (or reservoir), then through the generating plant back out into the environment. If the collection facility is unable to store the arriving water, then water is diverted past the generating plant directly to the environment, with consequent loss of electricity generation. In the irrigation system, water polluted with pesticides and/or fertilizers flows from farmland to a collection facility before treatment, and then to the environment. If the collection facility is too small, some polluted water flows directly into the environment. In the urban sewage system, sewage flows from the sewage system to a large primary treatment facility, then to a second facility and then back to the environment. Overflow of raw sewage directly into the environment occurs if the primary plant is too small.

The basic variables are as follows.

- $r(t)$ Volume flow rate from the watershed (m^3/s) or mass flow rate of pollutant (kg/s).
- $s(t)$ Volume of water (m^3) or mass of pollutant (kg) in the collection facility or primary treatment plant.
- $s_{\rm c}$ Maximum capacity of the collection facility or primary treatment plant, (m^3) or (kg).
- ρ Volume or mass treatment rate (m^3/s) or (kg/s) in the water reclamation plant or generating facility. We will assume the treatment rate ρ to be constant, on the assumption that the storage facility supplies water continuously at this rate.
- T Final time or duration of the rainy season(s).

The decision maker must choose the storage capacity $s_{\rm c}$ and the treatment or processing rate ρ so that the reliability against overflow from the storage unit directly to the environment is acceptable. The time-variation of the rate of drainage $r(t)$ from the watershed (or farmland or sewage system) is unknown, so $s_{\rm c}$ and ρ must be large enough but not wasteful. We will use the concept of robust reliability for choosing the storage volume and processing rate [3]. We will illustrate the method through five examples. In the first three examples, info-gap models of uncertainty are used. The sequence of info-gap models progresses from very simple to more complicated models, corresponding to increasing amounts of prior information. The last two examples demonstrate hybrid info-gap/probabilistic uncertainty models.

The method of robust reliability is based on a combination of three components.

1) The process model describes the dependence of the stored volume $s(t)$ upon the drainage-rate function $r(t)$.
2) The uncertainty model describes the uncertainty in the drainage-rate function. The uncertainty model can be probabilistic, info-gap, or a combination of both. For instance, time-series analysis is widely used for probabilistic modeling in hydrology [19], [21].
3) The failure criterion states that the stored volume cannot exceed the storage capacity.

These three elements are combined to assess the robustness, $\hat{\alpha}$, which is the greatest value of the uncertainty parameter (which is consistent with no-failure of the process). The decision maker must choose the smallest storage capacity $s_{\rm c}$ and treatment rate ρ, for which the corresponding robustness is acceptably large. The judgment of how large a robustness is depends on a subjective calibration of $\hat{\alpha}$.

The material balance in the storage facility is modeled as

$$\frac{ds}{dt} = r(t) - \rho, \quad s(0) = 0. \tag{6}$$

The watershed flows into the storage facility at rate $r(t)$, and the storage unit drains to the treatment facility at the maximum rate ρ all the time.

B. Example 1: Total Rainfall

Let us suppose that the drainage rate from the watershed is constrained only by the total rainfall (or the total quantity of sewage or pollutant) in the season. That is, we have no information with which to constrain the instantaneous drainage rate, except that it must be non-negative and be equal to the total seasonal precipitation. This very limited information about the drainage-rate uncertainty can be encoded in the following info-gap model:

$$\mathcal{U}(\alpha) = \left\{ r(t)\colon r(t) \geq 0 \text{ and } \int_0^T r(t)\, dt \leq \alpha \right\}, \quad \alpha \geq 0. \tag{7}$$

Thus, $\mathcal{U}(\alpha)$ is the set of all drainage-rate functions corresponding to an annual preciptation of no more than α given in m^3 or kg. The uncertainty parameter α is simply the total rainfall in the season, which also is not known, and can take any non-negative value. Thus, (7) is not a single set, but rather a family of nested convex nests, in which α, the uncertainty parameter, also determines the level of nesting. We note that the rate of variation of the drainage function is not constrained, and the drainage rate may fluctuate in time by small or by arbitrarily large amounts. The info-gap in (7) has been used extensively in the design and analysis of measurement systems [2].

The storage facility fails if, at any time, the stored volume (or mass) exceeds the capacity

$$s(t) > s_{\rm c}. \tag{8}$$

The robustness is the greatest value of the uncertainty parameter α, for which the system does not violate the failure criterion. In order to determine the robustness, we seek the maximum stored volume

$$\hat{s}(t) = \max_{r(t) \in \mathcal{U}(\alpha)} s(t) \tag{9}$$
$$= \alpha - \rho t \tag{10}$$

where (10) employs (6) and (7). The robustness of the storage facility is obtained by equating the greatest stored volume to the maximum capacity and then solving for the uncertainty parameter α

$$\max_{0 \leq t \leq T} \hat{s}(t) = s_{\rm c} \Rightarrow \hat{\alpha} = s_{\rm c}. \tag{11}$$

The robustness is precisely equal to the storage capacity and is independent of the water treatment rate.

C. Example 2: Deviation from Nominal Drainage Rate

The info-gap in (7) presumes extremely limited prior information about the uncertain drainage rate. Now consider somewhat more extensive prior information. Let $\overline{r}(t)$ be a known nominal drainage-rate function, and consider unknown and possibly severe transient fluctuations around $\overline{r}(t)$. The following cumulative energy-bound info-gap is a good representation of transient uncertainty in the drainage rate function

$$\mathcal{U}(\alpha, \overline{r}) = \left\{ r(t)\colon r(t) \geq 0 \text{ and } \int_0^T [r(t) - \overline{r}(t)]^2\, dt \leq \alpha^2 \right\}$$
$$\alpha \geq 0 \tag{12}$$

This info-gap $\mathcal{U}(\alpha, \overline{r})$ constrains the integrated squared deviation of the actual from the nominal drainage rate. This uncertainty model contains functions that deviate more or less steadily from the nominal function, as well as functions with transient but dramatic deviations. As before, this info-gap is a family of nested convex sets for $\alpha \geq 0$. (Envelope-bound info-gap models could be used also [3], [5].)

The solution of (6) can be expressed as

$$s(t) = -\rho t + \int_0^t [r(t) - \overline{r}(t)]\, d\tau + \int_0^t \overline{r}(\tau)\, d\tau. \tag{13}$$

Schwarz's inequality, followed by employing the bound on $r(t)$ in the info-gap of (12), shows that

$$\left(\int_0^t [r(\tau) - \overline{r}(\tau)]\, d\tau \right)^2 \leq \underbrace{\int_0^t [r(\tau) - \overline{r}(\tau)]^2\, d\tau}_{\leq \alpha^2} \int_0^t d\tau \leq \alpha^2 t. \tag{14}$$

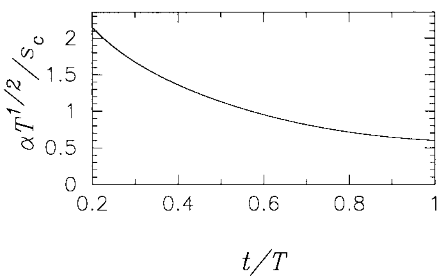

Fig. 4. Robustness versus time based on (19). $r_0 T/\pi s_c = 0.2$.

So combining (13) and (14), the greatest stored volume at time t is

$$\hat{s}(t) = \max_{r(t)\in\mathcal{U}(\alpha,\overline{r})} s(t) \tag{15}$$

$$= -\rho t + \int_0^t \overline{r}(\tau)\, d\tau + \alpha\sqrt{t}. \tag{16}$$

The robustness of the reservoir at time t is found by equating $\hat{s}$ to s_c and solving for α

$$\hat{s}(t) = s_c \Rightarrow \hat{\alpha}(t) = \frac{s_c - \int_0^t [\overline{r}(\tau) - \rho]\, d\tau}{\sqrt{t}}. \tag{17}$$

The overall robustness is

$$\hat{\alpha} = \min_{0\le t\le T} \hat{\alpha}(t). \tag{18}$$

We note that if $\overline{r} \ge \rho$, the instantaneous robustness $\hat{\alpha}(t)$ is monotonically decreasing in time. Thus, the minimum in (18) occurs at $t = T$. This is illustrated in Fig. 4, where $\overline{r}(t) = \rho + r_0 \sin \pi t/T$. In this case, the robustness as a function of time becomes

$$\hat{\alpha}(t) = \frac{s_c}{\sqrt{T}}\, \frac{1 - \dfrac{r_0 T}{\pi s_c}(1 - \cos \pi t/T)}{\sqrt{t/T}}. \tag{19}$$

The plot in Fig. 4 starts at $t/T = 0.2$ to avoid the singularity at the origin.

Returning to (17), we recognize that the instantaneous robustness $\hat{\alpha}(t)$ may vary quite substantially throughout the season for any choice of the design parameters s_c and ρ. This is illustrated in Fig. 4, where one sees that the robustness at the end of the season $(t/T = 1)$ is only about a quarter of the robustness at $t/T = 0.2$. In other words, the end of the season requires much greater storage and treatment resources than are required for the early part of the season. Conversely, the robustness of the facility-to-uncertain drainage-rate fluctuations is much lower at the end of the season than early in time. This analysis enables the decision maker to contemplate alternatives, such as allowing temporary violation of the no-spill requirement, in order to reduce the cost of the required facilities.

This example is sufficiently rich to illustrate that uncertainty may be either pernicious or propitious, as we have explained in Section III. That is, uncertain variations may be either adverse or favorable. The robustness $\hat{\alpha}(t)$ is the greatest value of the uncertainty parameter α such that the reservoir does not overflow. Equivalently, $\hat{\alpha}(t)$ is the greatest uncertainty consistent with no-failure and expresses the pernicious face of uncertainty. $\hat{\alpha}(t)$ will be large for a good design, implying large immunity to uncertain fluctuation, while a small value for $\hat{\alpha}(t)$ is undesirable.

In addition to wishing to prevent overflow of the reservoir, we may wish to use it for some secondary purpose for which the stored volume $s(t)$ must be substantially less than the actual reservoir capacity. We will denote our "wildest dream" of the maximum stored volume by s_w, where s_w is much less than s_c. The uncertain fluctuations in the flow rate $r(t)$ may be favorable and lead to stored volume no greater than s_w. The opportunity function $\hat{\alpha}(t)$ expresses the lowest level of uncertainty that is sufficient to entail the possibility of stored volume as low as s_w. A small value of $\hat{\beta}(t)$ is desirable, while a large value implies that great ambient uncertainty (with the attendant risks of flooding) is needed in order to achieve a very low stored volume.

The robustness function $\hat{\alpha}(t)$ was found by evaluating the maximum stored volume in (16) and equating this to the storage capacity s_c in (17). The opportunity function $\hat{\beta}(t)$ expresses the least uncertainty that must be tolerated in order to enable the possibility of stored volume as low as the value s_w. This is found by evaluating the minimum stored volume and equating this to the wildest-dream storage capacity s_w. One finds

$$\hat{\beta}(t) = \frac{-s_w + \int_0^t [\overline{r}(\tau) - \rho]\, d\tau}{\sqrt{t}}. \tag{20}$$

Comparing (17) and (20), one sees that the robustness and opportunity functions are mirror images of one another

$$\hat{\beta}(t) = -\hat{\alpha}(t) + \frac{s_c - s_w}{\sqrt{t}}. \tag{21}$$

We see that, as the robustness increases, the opportunity decreases and vice versa. Since "big is better" for $\hat{\alpha}$, while "big is bad" for $\hat{\beta}$, this means that $\hat{\alpha}$ and $\hat{\beta}$ act sympathetically. Any design-change that enhances the robustness also enhances the opportunity for successful secondary exploitation. This pleasing symbiosis is (unfortunately) not a universal attribute of ambient uncertainty.

D. Example 3: Bounded Variation of the Drainage Rate

Now consider a more informative info-gap for the uncertainty in the drainage-rate function $r(t)$. The previous models have included functions that quickly can vary arbitrarily. We now consider an ellipsoid-bound info-gap that constrains the unknown variation of $r(t)$ to a known range of frequencies. The known nominal-drainage rate is $\overline{r}(t)$, and the actual function deviates from $\overline{r}$ by a truncated Fourier series with uncertain coefficients. Represent the drainage rate function as

$$r(t) = \overline{r}(t) + \sum_{n=n_1}^{n_2} \beta_n \cos \frac{n\pi t}{T}. \tag{22}$$

Let β be the vector of Fourier coefficients and let $\gamma(t)$ represent the vector of cosine functions. Equation (22) can be rewritten more succinctly as

$$r(t) = \overline{r}(t) + \beta^T \gamma(t) \tag{23}$$

where the superscript T implies matrix transposition. The Fourier ellipsoid-bound uncertainty model for the drainage-rate function is

$$\mathcal{U}(\alpha, \overline{r}) = \{r(t) = \overline{r}(t) + \beta^T \gamma(t) \colon \beta^T W \beta \le \alpha^2\}, \quad \alpha \ge 0 \tag{24}$$

where W is a known, real, symmetric, positive, definite matrix. The expression $\beta^T W \beta \le \alpha^2$ defines an ellipsoidal region of β-vectors. The shape of the ellipsoid is fixed by the matrix W, while the size of the ellipsoid is determined by the uncertainty parameter α. So the vector β of unknown Fourier coefficients is constrained to an ellipsoid of uncertain size α.

Combining (23) with the solution of (6), the stored volume at time t can be written as

$$s(t) = \int_0^t [\overline{r}(\tau) - \rho] \, d\tau + \beta^T \underbrace{\int_0^t \gamma(\tau) \, d\tau}_{\zeta(t)} \tag{25}$$

which defines the known vector function $\zeta(t)$.

Using Lagrange optimization, the maximum of $\beta^T \zeta(t)$, as β varies over the info-gap $\mathcal{U}(\alpha, \overline{r})$, is found to be

$$\max_{\beta \in \mathcal{U}(\alpha, \overline{r})} \beta^T \zeta(t) = \alpha \sqrt{\zeta(t)^T W^{-1} \zeta(t)} \tag{26}$$

which occurs with the drainage function, whose Fourier coefficient vector is

$$\beta = \frac{\alpha}{\sqrt{\zeta(t)^T W^{-1} \zeta(t)}} W^{-1} \zeta(t). \tag{27}$$

So, the maximum stored volume at time t is

$$\hat{s}(t) = \max_{r \in \mathcal{U}(\alpha, \overline{r})} s(t) = \int_0^t [\overline{r}(\tau) - \rho] \, d\tau + \alpha \sqrt{\zeta(t)^T W^{-1} \zeta(t)}. \tag{28}$$

From this, the robustness at time t is found by equating $\hat{s}(t)$ to the storage capacity s_c and solving for the uncertainty parameter $\hat{\alpha}$, resulting in

$$\hat{\alpha}(t) = \frac{s_c - \displaystyle\int_0^t [\overline{r}(\tau) - \rho] \, d\tau}{\sqrt{\zeta(t)^T W^{-1} \zeta(t)}} \tag{29}$$

The overall robustness is found, as before, from (18).

As a specific example, let $n_1 = 1$ and $n_2 = N$ in (22), and let $W = I$ be the identity matrix in (24). The denominator in (29) now takes a simple form, and the instantaneous robustness becomes

$$\hat{\alpha}(t) = \frac{s_c - \displaystyle\int_0^t [\overline{r}(\tau) - \rho] \, d\tau}{\sqrt{\displaystyle\sum_{n=1}^{N} \left(\frac{T}{n\pi} \sin \frac{n\pi t}{T} \right)^2}}. \tag{30}$$

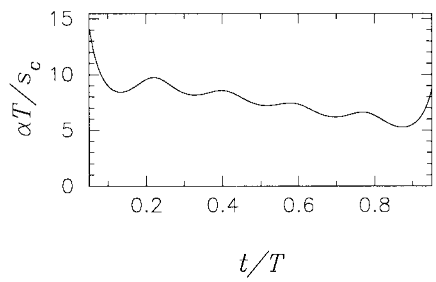

Fig. 5. Robustness versus time based on (31). $r_0 T / \pi s_c = 0.2$ and number of modes is $N = 5$.

As a further illustration, consider a simple expression for the nominal drainage rate $\overline{r}(t) - \rho = r_0 \sin \pi t / T$. The instantaneous robustness becomes

$$\hat{\alpha}(t) = \frac{s_c - \dfrac{r_0 T}{\pi} \left(1 - \cos \dfrac{\pi t}{T} \right)}{\sqrt{\displaystyle\sum_{n=1}^{N} \left(\frac{T}{n\pi} \sin \frac{n\pi t}{T} \right)^2}} \tag{31}$$

which is illustrated in Fig. 5. Again note that, as in the second example, the instantaneous robustness varies substantially throughout the season, implying that the no-spill requirement results in storage and treatment facilities that will operate below capacity for much of the time. In Fig. 5, the robustness varies by a factor of two throughout the season, without considering the singularities at the extremes ($t = 0$ and $t = T$).

E. Example 4: Modification of Example 1—Hybrid Uncertainty Model

Let us suppose that, in addition to the uncertainty model of (7), we also know the probability density function for the total seasonal rainfall: $p(\alpha)$ (recall that the unknown uncertainty parameter α in (7) is precisely the total seasonal precipitation). This can be folded into the analysis of Example 1 in the following way. In (11), we found that the greatest tolerable value of the uncertainty parameter α is simply the volumetric capacity s_c of the reservoir. The additional probabilistic information can be used to choose s_c so that the probability of failure is small enough (equal to ε, for example). That is, s_c is chosen as the upper $(1 - \varepsilon)$-quantile of the distribution of the precipitation

$$\int_{s_c}^{\infty} p(\alpha) \, d\alpha = \varepsilon. \tag{32}$$

The very sparse info-gap for the uncertainty in the time-variation of the rate of drainage given in (7) entails the possibility that all the seasonal precipitation reaches the reservoir at the same time. This information is augmented by the probabilistic uncertainty in the total precipitation, which is used to evaluate the likelihood that the reservoir will overflow.

F. Example 5: Modification of Example 2—Hybrid Uncertainty with Two Randomly Timed Storms

Let us suppose that the rainy season comprises exactly two storms and that they occur randomly in time. Let the length of the dry spell between the two storms be exponentially distributed, as would be expected if the times of occurrence of the storms have a Poisson distribution. Furthermore, the uncertainty in the time-variation of the drainage-rate function during a storm is described by the info-gap of (12). We are again considering a hybrid uncertainty model, which could be generalized to an arbitrary number of storms distributed randomly throughout the season.

Let θ be the duration of each of the two storms. Let us suppose that the nominal drainage-rate function during a storm is constant and exceeds ρ so that $\overline{r}(t)-\rho = c$. Then, employing (16), we find that the greatest stored volume after the first storm occurs at the end of the storm and equals

$$\hat{s}_1 = c\theta + \alpha\sqrt{\theta} \tag{33}$$

After the storm, the reservoir continues to drain through the treatment plant at the rate ρ. If the dry spell until the second storm is of duration Δ (which is a random variable), then the reservoir starts the second storm with a maximum water inventory of $\hat{s}_1 - \rho\Delta$. This initial inventory continues to drain in parallel to the water added by the second storm. So, the inventory at time t after the start of the second storm is

$$s_2(t) = \int_0^t [r(\tau) - \rho]\, d\tau + \hat{s}_1 - \rho(\Delta + t). \tag{34}$$

Hence, the maximum inventory at time t into the second storm, given maximum residual inventory from the first storm, is

$$\hat{s}_2(t) = ct + \alpha\sqrt{t} + \hat{s}_1 - \rho(\Delta + t). \tag{35}$$

We have adopted the conservative simplification that the residual inventory continues to drain (rather than drying up) at least until the end of the second storm. If ρ is sufficiently small, the maximum inventory occurs at the end of the second storm and is

$$\hat{s}_2 = c\theta + \alpha\sqrt{\theta} + \hat{s}_1 - \rho(\Delta + \theta). \tag{36}$$

Employing (33), this becomes

$$\hat{s}_2 = 2(c\theta + \alpha\sqrt{\theta}) - \rho(\Delta + \theta). \tag{37}$$

The robustness is evaluated by equating this maximal inventory to the reservoir capacity and solving for the uncertainty parameter

$$\hat{s}_2 = s_{\mathrm{c}} \Rightarrow \hat{\alpha}(\Delta) = \frac{s_{\mathrm{c}} + \rho(\Delta + \theta) - 2\theta}{2\sqrt{\theta}}. \tag{38}$$

We have written the robustness $\hat{\alpha}(\Delta)$ as an explicit function of the duration Δ of the dry spell, which is an exponential random variable, and we note that $\hat{\alpha}$ also depends on the reservoir capacity s_{c} and on the water-treatment rate ρ, which are the design variables. Since we know the probability distribution of Δ we can weight the robustness probabilistically as

$$\hat{\alpha} = \int_0^\infty \hat{\alpha}(\Delta) p(\Delta)\, d\Delta. \tag{39}$$

The duration of the dry spell is exponentially distributed, with density

$$p(\Delta) = \lambda e^{-\lambda\Delta}, \quad \Delta \geq 0 \tag{40}$$

(the infinite domain of $p(\Delta)$ is an approximation, but we are assuming that λ is large enough so that the statistical weight of dry spells longer than the entire season, and thus precluding two storms, is negligible). Employing (38), the randomized robustness in (39) becomes

$$\hat{\alpha} = \frac{s_{\mathrm{c}}}{2\sqrt{\theta}} + \frac{\rho\sqrt{\theta}}{2} - c\sqrt{\theta} + \frac{\rho}{2\lambda\sqrt{\theta}}. \tag{41}$$

This relation serves the decision maker in choosing the reservoir volume s_{c} and the water-treatment rate ρ to achieve acceptable robustness with respect to both the timing of the storms and their instantaneous temporal variability.

VI. Conclusions

Three fundamentally distinct approaches are available for formally describing uncertainty: probability, fuzzy set theory, and information-gap modeling. We have concentrated on the latter and have illustrated decision making in watershed-management problems. As stressed in the paper, info-gap modeling is especially useful for systematically studying highly uncertain situations. By acknowledging the presence of uncertainty and attempting to describe it as realistically as possible, decision makers should be able to make wiser and more realistic decisions when confronting real-world decision problems.

The idealistic goal of achieving sustainable development is fraught with a range of baffling uncertainties arising within the natural world and society and their complex interactions. As a matter of fact, authors such as Falkenmark [13] and Kundzewicz [24] point out that it may be almost impossible to attain sustainable development with respect to the availability of water in adequate quantity and quality. The ongoing population explosion, coupled with water shortages and the widespread pollution of surface and groundwater supplies, may mean that the ability of future generations to meet their own water needs has been severely compromised already. The foregoing and other alarming trends can only exacerbate the depth and breadth of uncertainty inherent in sustainable development and emphasize the great importance of having formal modeling procedures for better comprehending uncertainty. In addition, with regard to sustainable water management, planning components should include carrying out integrated societal/land/water planning at the river basin level, developing suitable policies to permit comprehensive management, constructing water resources retrieval and conveyance systems to make water readily available to society, and controlling sources of pollution [13].

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