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Info-gap Robust Design with Load and Model Uncertainties

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Contents

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Abstract

This paper develops a new structural design concept which incorporates uncertainties in both the load and the structural model parameters. Info-gap models of uncertainty are used to represent uncertainty in the power spectral density of the load and in parameters of the vibration model of the structure. It is demonstrated that any design which optimizes functional performance will also minimize the robustness to uncertainty. Since uncertainties are prevalent in many applications, this paper argues that it is necessary to *satisfy critical performance requirements* (rather than to optimize performance), and to *maximize the robustness to uncertainty.* The design implications of this *robust-satisficing* approach are demonstrated with several heuristic structural design examples. It is shown that design preferences depend upon performance requirements: preferences between designs can be reversed when performance requirements change. Also, we show that the info-gap robustness function provides an attractive tool for adjudicating between conflicting objectives in multi-criteria design.

1 Introduction

Design of civil structures in seismically active regions requires consideration of both the uncertainty in earthquake ground motions and the uncertainty of design-base structural models. This problem is challenging and only a limited number of publications deal with both uncertainties, for example [6–11]. Because civil structures are not mass-produced and because the occurrence rate of large earthquakes is very low, the probabilistic representation of these uncertainties seems to be difficult in most cases. To deal with the severely deficient information about large earthquakes and their uncertain dynamic interactions with structures, info-gap decision theory [1, 2, 4] is employed in the present paper.

The purpose of this paper is to propose a new structural design concept which incorporates uncertainties in both the load and the structural model parameters. For that purpose, it is necessary to identify the critical load (or excitation) and the corresponding critical set of structural model parameters. It is clear that the critical load (or excitation) depends on the structural model parameters and it is extremely difficult to deal with load and structural model parameter uncertainties simultaneously. Throughout the paper, uncertainty in structural model parameters is represented with info-gap models of uncertainty [1, 2, 4]. In sections 5 and 6 the load uncertainty is tentatively removed by adopting the most critical case [14, 15], i.e. a rectangular critical power spectral density of the input ground acceleration. This entails assuming knowledge of an upper bound of the power spectral density (PSD) of the input ground acceleration. This enables simple and fairly accurate calculation of the critical PSD shape. However, in fact one rarely knows a realistic upper bound of the PSD. In sections 7 and 8 this assumption is removed and an info-gap model is used to represent uncertainty in the upper bound of the PSD.

A central theoretical assertion of this paper is that any design which optimizes functional performance will also minimize the robustness to uncertainty. In other words, there is an irrevocable trade-off between the functional performance of a structure and its immunity to uncertainties in the knowledge upon which the design is based. Since uncertainties are prevalent and potent in many applications, this paper argues that it is necessary to **satisfy critical performance requirements** (rather than to optimize performance), and to **maximize the robustness to uncertainty.** We refer to this design strategy as **robust-satisficing.**

An important consequence of robust-satisficing is that a design which is preferred at one level of performance, may be superseded by a different design if a different level of performance is required. This **preference reversal** is rare when performance is optimized, but quite common when performance is satisficed and robustness is optimized, as we will see.

Many methodologies have been developed for managing uncertainty in the analysis and design of engineering structures. This includes parametric, non-parametric, Bayesian and other probabilistic tools, as well as axiomatically distinct theoretical constructs such as fuzzy logic. Info-gap methods can be combined with these approaches [4]. For discussion of the foundational distinctions between info-gap and probabilistic approaches, see [1–3].

The robustness function of info-gap theory is previewed in section 2, and the relation between robustness to uncertainty and functional performance is described. These issues are then examined in more detail in sections 3 and 4. Three illustrative examples are discussed in section 5 to help the reader to understand the proposed new design concept. The first example is a heavily-damped single-degree-of-freedom (SDOF) system subjected to a stationary random base acceleration, the second is a lightly-damped low-frequency SDOF system, and the third is a two-story shear building subjected to a stationary random base acceleration. The relation between the robust-satisficing optimal design and performance optimal design is explained in section 6. Finally, in section 7, the presence of load uncertainty is integrated with model-parameter uncertainty in the design methodology. This extended methodology is illustrated in section 8 with the analysis of a 6-story shear building. Our conclusions are summarized in section 9.

2 Preview of Info-gap Robustness Analysis

In this section we briefly summarize the basic design-approach derived from info-gap decision theory [1, 2, 4].

Design parameters are specified by a real-valued scalar or vector δ. These design parameters, such as the story stiffnesses, need to be chosen to assure functionality and reliability. Denote the performance (or cost) function by $\widehat{f}(\delta)$, for which a small value is desirable. The design must satisfy a critical performance requirement for $\widehat{f}(\delta)$:

$$\widehat{f}(\delta) \le f_{\rm c} \tag{1}$$

where $f_{\rm c}$ is the greatest acceptable value of the performance function. (Multiple performance requirements can also be imposed, as we will see in section 4.) The value of $f_{\rm c}$ is usually not chosen before the analysis, but arises from the iterative analysis of robustness to uncertainty, functional performance, and auxiliary constraints such as cost.

Relation (1) is called a **satisficing** performance requirement, as distinguished from an optimizing performance requirement where δ is chosen to minimize $\widehat{f}(\delta)$. 'Satisficing' means "To decide on and pursue a course of action that will satisfy the minimum requirements necessary to achieve a particular goal." [12]. We will discuss the relation between satisficing and optimizing performance requirements in section 6.

The performance function $\widehat{f}(\delta)$ depends on the dynamic properties of the structure, for example, through a transfer function $F(\omega, \delta)$. We will denote this by $\widehat{f}(F, \delta)$. The designer may use the best available models, but phenomena such as damping and cracking are extraordinarily complicated and those best models are likely to be incorrect in some unknown ways. Let $\widetilde{F}(\omega, \delta)$ denote our best-estimated model of the dynamic behavior of the structure.

Design is based on the nominal or best-estimated model $\widetilde{F}(\omega, \delta)$. The actual model is an unknown model $F(\omega, \delta)$. The basic **robustness question** is: how wrong can our model be, (how much can $\widetilde{F}$ differ from F), without jeopardizing the performance of the system? The answer to this question is the info-gap robustness function, $\widehat{\alpha}(\delta, f_{\rm c})$, which we will describe in section 4. The value of the robustness function depends on the choice of the design variables, δ, and on the performance specification $f_{\rm c}$. One chooses the design to **maximize the robustness** and to **satisfice the performance.** The robust-satisficing optimal design is the value of δ which maximizes $\widehat{\alpha}(\delta, f_{\rm c})$:

$$\widehat{\delta}(f_{\rm c}) = \arg\max_{\delta} \widehat{\alpha}(\delta, f_{\rm c}) \tag{2}$$

In the remainder of this paper we describe these ideas in more detail and develop several novel implications.

3 Info-gap Uncertainty Models

In this section we very briefly describe a few info-gap models of uncertainty in the dynamic model $F(\omega, \delta)$. In the specific problems we will consider, an info-gap model describes, non-probabilistically, the uncertain difference between the best-estimated model $\widetilde{F}(\omega, \delta)$ and other possible models $F(\omega, \delta)$.

An **info-gap model** for the uncertainty in the dynamic behavior of the structure is an unbounded family of nested sets of dynamic models. For instance, the **uniform-bound info-gap model** is:

$$\mathcal{F}(\alpha, \widetilde{F}) = \left\{ F(\omega, \delta) : \ |F(\omega, \delta) - \widetilde{F}(\omega, \delta)| \le \alpha \right\}, \quad \alpha \ge 0 \tag{3}$$

An info-gap model is an unbounded family of nested sets of models, rather than a single bounded set. Consequently, two levels of uncertainty are entailed in an info-gap model. First, for a given value of α, $\mathcal{F}(\alpha, \widetilde{F})$ is a set of possible dynamic models; which model is correct is unknown. The second level of uncertainty is the unknown value of α. As α gets larger, the range of possible models increases, so α is the unknown **horizon of uncertainty.**

It must be stressed that an info-gap model of uncertainty is **not** a single bounded set. Rather, an info-gap model is an unbounded family of nested sets. For any given value of α we have a single set, $\mathcal{F}(\alpha, \widetilde{F})$, of possible models. But since α is the unknown horizon of uncertainty, we have an entire family of uncertainty sets which become more and more inclusive as α gets larger. The unbounded uncertainty of an info-gap model is central to the present info-gap reliability analysis, and is rather different from the 'convex model' idea of a single bounded set of possibilities [5].

A common variation on the info-gap model of eq.(3) is the **envelope-bound info-gap model,** in which the possible variation of the models is constrained to an envelope of known shape but unknown size:

$$\mathcal{F}(\alpha, \widetilde{F}) = \left\{ F(\omega, \delta) : \ |F(\omega, \delta) - \widetilde{F}(\omega, \delta)| \le \alpha\psi(\omega) \right\}, \quad \alpha \ge 0 \tag{4}$$

The sets in both the uniform-bound and the envelope-bound info-gap model include a very rich range of functions. In particular, the info-gap models of eqs.(3) and (4) include functions of unbounded variation: functions $F(\omega, \delta)$ which have many peaks and which fluctuate wildly with frequency. This may very well be unrealistic. The designer may have information which constrains the rate of variation of $F(\omega, \delta)$ with frequency. There are several common info-gap models which incorporate information about constrained rate of variation [2]. One particularly simple example is the **parameter-uncertainty** info-gap model:

$$\mathcal{F}(\alpha, \widetilde{F}) = \left\{ F(\omega, c, \delta) : \ \left| \frac{c_i - \widetilde{c}_i}{\widetilde{c}_i} \right| \le \alpha, \ \ i = 1, \ldots, J \right\}, \quad \alpha \ge 0 \tag{5}$$

where c is the vector of uncertain parameters, such as damping coefficients, and $\widetilde{c}$ is the known vector of best-estimates of c upon which the best-estimated model $\widetilde{F}$ is based. The form of the transfer function $F(\omega, c, \delta)$ is known; only its parameters c are uncertain. This is different from the info-gap models of eqs.(3) and (4) which contain transfer functions of many different functional forms.

4 The Info-gap Robustness Function

In the previous section we formulated several typical info-gap models for uncertainty in the dynamic model of the structure. We can now formulate the info-gap answer to the robustness question, which was: how much error in the nominal design-base dynamic model is consistent with acceptable performance? The info-gap answer is expressed in the robustness function.

There are always three elements underlying a robustness function: an **uncertainty model**, a **system model**, and **performance requirements**. We will use an info-gap model for the uncertainty model, $\mathcal{F}(\alpha, \widetilde{F})$, $\alpha \ge 0$. The system model expresses the response, or "output", or dynamical behavior of the system. We will denote the system model by $\widehat{f}(F, \delta)$, where δ specifies the design variables and F represents the dynamic model upon which the design is based. $\widehat{f}(F, \delta)$ may be a vector of outputs: various different responses, for instance at different points. As in eq.(1), the performance requirements can be expressed as inequalities:

$$\widehat{f}_i(F, \delta) \le f_{\mathrm{c},i}, \quad i = 1, \ldots, R \tag{6}$$

The ith response must be no greater than the corresponding critical value $f_{\mathrm{c},i}$, for each $i = 1, \ldots, R$.

The robustness of design δ, to uncertainty in the dynamics, is the greatest horizon of uncertainty α up to which the response to all dynamic models $F(\omega, \delta)$ is acceptable according to each of the performance requirements:

$$\widehat{\alpha}(\delta, f_{\mathrm{c}}) = \max \left\{ \alpha : \left[\max_{F \in \mathcal{F}(\alpha, \widetilde{F})} \widehat{f}_i(F, \delta) \right] \le f_{\mathrm{c},i}, \text{ for all } i = 1, \ldots, R \right\} \tag{7}$$

The next section presents three examples of the robustness function and its uses.

5 Illustrative Examples

5.1 Heavily Damped SDOF System

We consider a heavily damped SDOF system subjected to stationary random acceleration loading. We will discuss the info-gap robustness curve and its design implications. In particular, we will emphasize the significance of the intersection of robustness curves.

5.1.1 Formulation

The performance function explicitly includes the cost of the stiffness and damping, as well as the variance of the displacement. The performance function is:

$$\widehat{f}(c, k) = 2\overline{s} \int_{\omega_L(k)}^{\omega_U(k)} F(\omega, c, k)\, \mathrm{d}\omega + 1.67 \times 10^{-8} k c^2 \quad \mathrm{m}^2 \tag{8}$$

The integral represents the variance of the displacement as explained in [14, 15], while the second term on the right expresses the cost of stiffness and damping. The units of the stiffness and damping coefficients, k and c, in eq.(8) are N/m and N·s/m, respectively. This stiffness-damping cost function is a simple heuristic example of empirical cost functions. $F(\omega, c, k) = m^2/[(k - \omega^2 m)^2 + (\omega c)^2]$ is the square of the transfer function, which we will refer to subsequently simply as the transfer function. $\overline{s} = 0.0661$ $\mathrm{m}^2/\mathrm{s}^3$ is the upper bound of the spectral density of the input base acceleration. The critical spectral density is rectangular with width $\Delta\omega = 0.553/(2\overline{s})$ rad/s. The limits of the integral are $\omega_{L,U}(k) = \Omega \mp \Delta\omega/2$ where $\Omega^2 = k/m$. The design variable is the stiffness k, and the damping coefficient c is uncertain.

The damping coefficient c is highly uncertain, and only a nominal design-base value $\widetilde{c}$ is known. We will represent the damping uncertainty with an info-gap model like eq.(5):

$$\mathcal{F}(\alpha, \widetilde{F}) = \left\{ F(\omega, c, k) : \left| \frac{c - \widetilde{c}}{\widetilde{c}} \right| \le \alpha \right\}, \quad \alpha \ge 0 \tag{9}$$

For any value of α, the unknown horizon of uncertainty, $\mathcal{F}(\alpha, \widetilde{F})$ is the set of transfer functions for which the damping coefficient differs fractionally from $\widetilde{c}$ by no more than α. Since α is unknown, the info-gap model is an unbounded family of nested sets of transfer functions.

5.1.2 Trade-off Between Performance and Robustness to Uncertainty

A small value is desired for the performance function $\widehat{f}(c, k)$; let f_{c} denote an acceptably small value. At this stage of the design-analysis we treat f_{c} as a free parameter, or at best we tentatively adopt a value of f_c, since the final selection will depend on the trade-off between performance and robustness-to-uncertainty.

The robustness to damping-uncertainty, of design k, is the greatest horizon of uncertainty, α, up to which all realizations of $F(\omega, c, k)$ result in acceptable performance:

$$\widehat{\alpha}(k, f_c) = \max \left\{ \alpha : \left[\max_{F \in \mathcal{F}(\alpha, \widetilde{F})} \widehat{f}(c, k) \right] \leq f_c \right\} \tag{10}$$

We can "read" this expression from left to right: The robustness $\widehat{\alpha}(k, f_c)$, of design k with performance specification f_c, is the maximum horizon of uncertainty α up to which all transfer functions $F(\omega, c, k) \in \mathcal{F}(\alpha, \widetilde{F})$, result in performance $\widehat{f}(c, k)$ which is no worse than f_c. A large value of $\widehat{\alpha}(k, f_c)$ means that the design is highly immune to uncertainty in the damping, while a small value of $\widehat{\alpha}(k, f_c)$ implies that the performance of the system is highly vulnerable to error in the damping coefficient. The system can be relied upon to perform as required when $\widehat{\alpha}(k, f_c)$ is large; the system is unreliable when $\widehat{\alpha}(k, f_c)$ is small.

The robustness function of eq.(10) is shown in fig. 1 for several values of the design variable, k. The mass is normalized to unity, i.e. 1kg. The nominal damping coefficient $\widetilde{c}$ is 4.9 which corresponds to a damping ratio of 0.20 for $k = 158$.

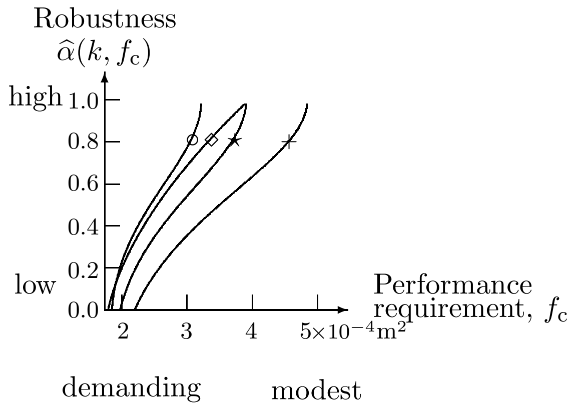

Figure 1: Robustness function, eq.(10), for several values of stiffness k: 158 (○), 1.44×158 (◇), 0.81×158 (⋆), 0.64×158 (+).

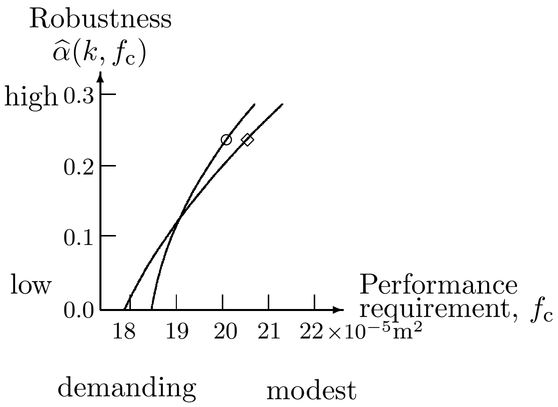

Figure 2: Robustness function, eq.(10), for two values of stiffness k: 158 (○), 1.44×158 (◇). Expanded from fig. 1.

The first thing to note in fig. 1 is that the robustness improves ($\widehat{\alpha}(k, f_c)$ gets larger) as the performance requirement is relaxed (f_c is increased). This is the unavoidable trade-off between robustness-to-uncertainty in the damping, and functional performance: any improvement in the demanded performance can be obtained only by relinquishing immunity to uncertainty in the damping coefficient. This trade-off is a ubiquitous property of the info-gap robustness function [2].

Another point to note in fig. 1 is that, for each value of k, the robustness becomes zero for some value of f_c. For instance, $\widehat{\alpha}(k, f_c) = 0$ at $k = 0.64 \times 158$ and $f_c = 2.2 \times 10^{-4}$. This means that, for this design, arbitrarily small error in the design-base value of the damping coefficient $\widetilde{c}$, can result in violation of the performance requirement with this value of f_c. In other words, this design cannot be relied upon to perform as well as this value of f_c.

What level of performance *can* be relied upon with this design ($k = 0.64 \times 158$)? Moving 'uphill' on the robustness curve marked '+', we find that $\widehat{\alpha}(k, f_c) = 0.2$ at $f_c = 2.6 \times 10^{-4}$. Referring to the info-gap model of eq.(9) we see that a robustness of 0.2 implies that the structure is immune against $\pm 20\%$ fractional error in the damping coefficient. Since the robustness curve increases monotonically, further improvement in robustness can be obtained by further relaxation of the performance requirement.

It is important to understand what value of f_c causes the robustness to become zero. Consideration of the definition of the robustness function in eq.(10) reveals that the robustness is zero when the demanded performance equals the design-base value:

$$\widehat{\alpha}(k, f_{\rm c}) = 0 \quad \text{if} \quad f_{\rm c} = \widehat{f}(\widetilde{c}, k) \tag{11}$$

$\widetilde{c}$ is the known, design-base value of the damping coefficient, and $\widehat{f}(\widetilde{c}, k)$ is the value of the performance function based on this best-estimate of c. Eq.(11) asserts that the anticipated best-estimate of performance, of any design k, cannot be relied upon to occur. Further implications of relations such as eq.(11) are discussed in section 6.

The final point to note from fig. 1 concerns the comparison of alternative designs. The robustness curve of design $k_\star = 0.81 \times 158$ (marked with a '$\star$') hits the horizontal axis at $f_{{\rm c}\star} = 2.0 \times 10^{-4}$. This is a more demanding performance requirement than $f_{{\rm c}+} = 2.2 \times 10^{-4}$, which is the value at which design $k_+ = 0.64 \times 158$ (marked with a '+') loses all robustness. In other words, in terms of the nominal design-base anticipation of performance, $k_\star$ is better than k_+. However, this is an unrealistic comparison, since neither of these designs can be relied upon to perform as well as these values of $f_{{\rm c}\star}$ and $f_{{\rm c}+}$ indicate, since the respective robustnesses are zero.

The proper comparison of designs $k_\star$ and k_+ is at positive values of robustness. We see from the robustness curves '$\star$' and '+' in fig. 1 that $\widehat{\alpha}(k_\star, f_{\rm c}) > \widehat{\alpha}(k_+, f_{\rm c})$ at all levels of performance appearing in the figure. In fact, the excess robustness of $k_\star$ is substantial. For example, at $f_{\rm c} = 3 \times 10^{-4}$ we see that $\widehat{\alpha}(k_\star, f_{\rm c}) = 0.54$ while $\widehat{\alpha}(k_+, f_{\rm c}) = 0.35$, meaning that design $k_\star$ is immune to $\pm 19\%$ more variation of damping than design k_+. From this we conclude that $k_\star$ is a more reliable, and hence a more desirable, design than k_+.

Fig. 2 shows an expanded view of part of two robustness curves appearing in fig. 1. The curves in fig. 2 show an important and not uncommon phenomenon: intersection of robustness curves. From our discussion of designs $k_\star$ and k_+ in fig. 1, we know that the preference between designs depends on the relative positions of the corresponding robustness curves. In fig. 2 we see the important phenomenon of **preference reversal.** At more demanding levels of performance ($f_{\rm c} < 19 \times 10^{-5}$), design $k_\diamond$ is preferred over $k_\circ$, while at larger (less demanding) values of $f_{\rm c}$ the designer's preference is reversed. Once again we see the importance of comparing designs at *positive* values of robustness, rather than along the axis of $\widehat{\alpha} = 0$.

5.2 Lightly-Damped Low-Frequency SDOF System

Consider a simple stylized design problem for a linear SDOF system. The stiffness must be chosen to obtain low damping and low natural frequency. The performance function is:

$$\widehat{f}(k) = \gamma^2 \zeta^2 + \Omega^2 \tag{12}$$

where $\zeta = \widetilde{c}/(2\sqrt{mk})$ is the damping ratio, $\Omega = \sqrt{k/m}$ is the natural frequency, m is the mass, $\widetilde{c}$ is the best estimate of the damping coefficient, and γ is a positive constant which expresses the importance of damping compared to natural frequency. We begin by supposing that m and $\widetilde{c}$ are known. We will find the stiffness, $k^\star$, which minimizes the performance function. We will then consider the damping coefficient to be uncertain, and formulate an info-gap model and the robustness function. We will find the stiffness, $\widehat{k}$, which satisfices the performance and maximizes the robustness. We will see that this robust-satisficing stiffness is more robust than the performance-minimizing stiffness for all positive values of robustness.

5.2.1 Performance and Robustness

The performance-optimizing stiffness, which minimizes the performance function $\widehat{f}(k)$ of eq.(12), is found by solving $0 = \partial \widehat{f}/\partial k$ for k, resulting in:

$$k^\star = \frac{\gamma \widetilde{c}}{2} \tag{13}$$

Damping coefficients are proverbially difficult to measure, so consider the damping coefficient to be highly uncertain. Let $\widetilde{c}$ be the best estimate or nominal value of the damping coefficient, and suppose the fractional error between this estimate, and the correct value c, is unknown. In this case an info-gap model like eq.(5) is appropriate:

$$\mathcal{F}(\alpha, \widetilde{c}) = \left\{ c : \left| \frac{c - \widetilde{c}}{\widetilde{c}} \right| \leq \alpha \right\}, \quad \alpha \geq 0 \tag{14}$$

Let f_c be the largest acceptable value of the performance function, $\widehat{f}(k, c)$, where $\widehat{f}(k, c)$ is eq.(12) with $\widetilde{c}$ replaced by c. (Note that $\widetilde{c}$ is now the centerpoint of the info-gap model $\mathcal{F}(\alpha, \widetilde{c})$.) The robustness to uncertainty, of the design with stiffness k, is the greatest horizon of uncertainty, α, up to which the performance is acceptable for all realizations of the damping c:

$$\widehat{\alpha}(k, f_c) = \max \left\{ \alpha : \left[\max_{c \in \mathcal{F}(\alpha, \widetilde{c})} \widehat{f}(k, c) \right] \leq f_c \right\} \tag{15}$$

$\widehat{\alpha}(k, f_c)$ is derived by first using eq.(12) to obtain an explicit expression for the inner maximum in eq.(15), resulting in:

$$\max_{c \in \mathcal{F}(\alpha, \widetilde{c})} \widehat{f}(k, c) = \frac{\gamma^2 \widetilde{c}^2 (1 + \alpha)^2}{4mk} + \frac{k}{m} \tag{16}$$

This is then equated to f_c and solved for α to find the robustness function:

$$\widehat{\alpha}(k, f_c) = \frac{\sqrt{[f_c - (k/m)]4mk}}{\gamma \widetilde{c}} - 1 \tag{17}$$

if this expression is non-negative; $\widehat{\alpha}(k, f_c) = 0$ otherwise which occurs if $\widehat{f}(k, \widetilde{c}) > f_c$.

5.2.2 Trade-off Between Performance and Robustness

The first thing to note about eq.(17) is that the robustness improves ($\widehat{\alpha}(k, f_c)$ increases) as the performance requirement becomes less stringent (f_c gets larger). This expresses the inevitable trade-off between robustness-to-uncertainty and functional performance which was noted in fig. 1: either one can be improved only by degrading the other.

Furthermore, for any design, k, the robustness of eq.(17) is zero for all performance requirements less than or equal to:

$$f_0 = \frac{\gamma^2 \widetilde{c}^2}{4mk} + \frac{k}{m} \tag{18}$$

We can evaluate f_0 at the performance-optimal stiffness $k^\star$ of eq.(13). Likewise, we can evaluate the performance function $\widehat{f}(k, \widetilde{c})$ at $k^\star$. We find these expressions are equal:

$$f_0(k^\star) = \widehat{f}(k^\star, \widetilde{c}) = \frac{\gamma \widetilde{c}}{m} \tag{19}$$

In other words, we have found that the robustness-to-uncertainty in the damping, of the performance-optimal design $k^\star$, vanishes at the performance-optimum:

$$\widehat{\alpha}(k^\star, f_c) = 0 \quad \text{if} \quad f_c = \widehat{f}(k^\star, \widetilde{c}) \tag{20}$$

The performance-optimal design $k^\star$ was found by using the best-estimate of the damping coefficient $\widetilde{c}$. However, the damping coefficient is highly uncertain. Eq.(20) shows that arbitrarily small error in $\widetilde{c}$ can cause violation of the performance requirement, when using the performance-optimal design $k^\star$ and imposing the corresponding minimal performance requirement $\widehat{f}(k^\star)$. We cannot rely on obtaining the performance which is anticipated from $k^\star$ because the robustness to uncertainty is zero.

5.2.3 Optimal Robust-satisficing Design

Now we seek the stiffness which maximizes the robustness. $\widehat{\alpha}(k, f_c)$ in eq.(17) has a single maximum at:

$$\widehat{k}(f_c) = \frac{m f_c}{2} \tag{21}$$

This optimal robust-satisficing stiffness decreases as the design requirement, f_c, becomes more stringent (smaller).

We now compare the robustness of the performance-optimizing design, $k^\star$ in eq.(13), and the optimal robust-satisficing design, $\widehat{k}(f_c)$ in eq.(21), by substituting first $k^\star$ and then $\widehat{k}(f_c)$ into $\widehat{\alpha}(k, f_c)$ of eq.(17):

$$\widehat{\alpha}(k^\star, f_c) = -1 + \sqrt{\frac{2 m f_c}{\gamma \widetilde{c}} - 1} \tag{22}$$

$$\widehat{\alpha}(\widehat{k}(f_c),\, f_c) = -1 + \frac{f_c m}{\gamma \widetilde{c}} \tag{23}$$

provided $f_c \geq \gamma \widetilde{c}/m$; both robustnesses vanish for lower values of f_c. It is readily seen that:

$$\widehat{\alpha}(\widehat{k}(f_c),\, f_c) \geq \widehat{\alpha}(k^\star, f_c) \tag{24}$$

with equality only when both robustnesses equal zero. Eqs.(22) and (23) are illustrated in fig. 3.

Fig. 3 demonstrates, as noted already in eqs.(19) and (20), that the robustness-to-uncertainty of the performance-optimal design $k^\star$ is zero at the optimal performance requirement $f_c = \gamma \widetilde{c}/m$. This means that only sub-optimal performance is reliable: $\widehat{\alpha} > 0$ only for $f_c > \gamma \widetilde{c}/m$. Finally, the info-gap robust-satisficing design $\widehat{k}(f_c)$ is more robust than the performance-optimal design $k^\star$ at any reliable performance requirement as indicated in eq.(24). The practical significance of this is that design $\widehat{k}(f_c)$ is preferred to design $k^\star$.

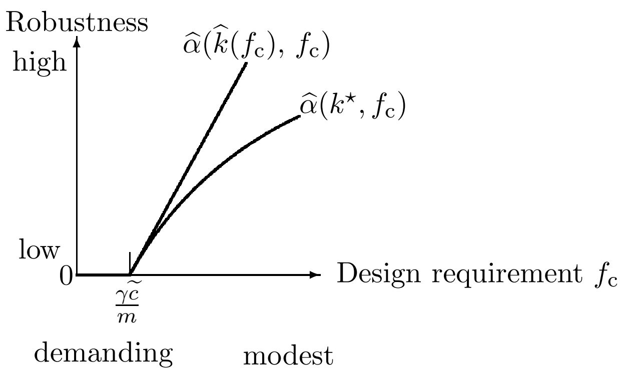

Figure 3: Illustration of eqs.(22) and (23), demonstrating the preference for optimal robust-satisficing design $\widehat{k}(f_c)$ over the performance-optimizing design $k^\star$.

5.3 2-Story Shear Building

We consider the design of a 2-story shear building with linear vibration dynamics and subjected to stationary random ground acceleration. We will use info-gap analysis to evaluate the trade-off, between performance and robustness-to-uncertainty, which is entailed in changing the cost of the building stiffness.

5.3.1 Formulation

The linear vibration model is described in [15]. The story stiffnesses, $\mathbf{k} = \{k_1, k_2\}$, are the design variables. The design has two aims. The first aim is to reduce the variances of the interstory drifts. The second aim is to enhance the robustness to uncertainty in the damping coefficients of the structure.

It has been shown [14, 15] that, to a good approximation, the critical excitation spectral density will be rectangular, and that the sum of the variances of the interstory drifts can be expressed as:

$$\widehat{f}(\mathbf{c}, \mathbf{k}) = 2\overline{s} \int_{\omega_L(\mathbf{k})}^{\omega_U(\mathbf{k})} F(\omega, \mathbf{c}, \mathbf{k}) \, \mathrm{d}\omega \tag{25}$$

where $F(\omega, \mathbf{c}, \mathbf{k})$ is the the sum of the transfer functions squared, which depends upon the properties of the building, in particular, the design variables $\mathbf{k}$ and the damping coefficients $\mathbf{c} = \{c_1, c_2\}$. $\overline{s}$ is an upper bound of the spectral density.

The damping coefficients for the two stories are highly uncertain, and only nominal design-base values $\widetilde{c}_1$ and $\widetilde{c}_2$ are known. We will represent the damping uncertainty with an info-gap model like eq.(5):

$$\mathcal{F}(\alpha, \widetilde{F}) = \left\{ F(\omega, \mathbf{c}, \mathbf{k}) : \ \left| \frac{c_i - \widetilde{c}_i}{\widetilde{c}_i} \right| \leq \alpha, \quad i = 1, \, 2 \right\}, \quad \alpha \geq 0 \tag{26}$$

The form of the transfer function $F(\omega, \mathbf{c}, \mathbf{k})$ is known and described in [15], while the damping coefficient vector $\mathbf{c}$ varies uncertainly from the known nominal design-base vector $\widetilde{\mathbf{c}}$.

The design constraint is that the sum of the normalized story stiffnesses is fixed, $k_1 + k_2 = K$, which approximates a cost or mass constraint. The floor mass is 32.0×10^3 kg in each floor. The apportionment of stiffness between the two stories which minimizes the performance function $\widehat{f}(\mathbf{c}, \mathbf{k})$ is close to the frequency constrained optimal design $k_1^\star = 3k_2^\star/2$, where k_1 is the stiffness of the lower story [15]. This frequency constrained optimum will be referred to as the performance optimal design.

5.3.2 Trade-off Between Robustness, Performance and Weight

Let f_{c} denote an acceptable value of the performance function $\widehat{f}(\mathbf{c}, \mathbf{k})$. The robustness to damping-uncertainty, of design $\mathbf{k}$, is the greatest horizon of uncertainty, α, up to which all realizations of $F(\omega, \mathbf{c}, \mathbf{k})$ result in acceptable performance:

$$\widehat{\alpha}(\mathbf{k}, f_{\mathrm{c}}) = \max \left\{ \alpha : \ \left[\max_{F \in \mathcal{F}(\alpha, \widetilde{F})} \widehat{f}(\mathbf{c}, \mathbf{k}) \right] \leq f_{\mathrm{c}} \right\} \tag{27}$$

The robustness function has been evaluated for the performance-optimizing stiffness, $\mathbf{k}^\star$, and for two choices of the normalized total stiffness: $K = 4$ and $K = 5$. The fundamental natural period of design $K = 5$ is 0.5 s. The nominal damping coefficient is 1.01×10^5 N·s/m in each story for both designs, $K = 4$ and $K = 5$. These robustness curves are shown in fig. 4.

The numerical values of robustness are interpreted as follows. Referring to the info-gap model of eq.(26) we see that the horizon of uncertainty, α, has the meaning of fractional error in the damping coefficients. Hence, robustness of $\widehat{\alpha}(\mathbf{k}^\star, f_{\mathrm{c}}) = 0.3$ means that the performance is guaranteed to be no worse than f_{c} if the design-base damping coefficients $\widetilde{c}_i$ err by no more than ±30%.

The performance requirement f_{c} is the sum of the variances of the interstory drift, in units of m^2.

We note the usual trade-off between robustness and performance: $\widehat{\alpha}(k^\star, f_{\mathrm{c}})$ gets better (larger) as the demanded performance f_{c} gets worse (larger).

Another important use of the robustness curves in fig. 4 is in exploring the trade-off between weight (or cost) of the stiffness, and robustness to uncertainty in the damping. The '$K = 5$' curve lies above the '$K = 4$' curve in fig. 4, which means that more massive stiffness entails greater robustness to uncertainty, at the same level of performance. The vertical distance from the '$K = 4$' curve to the '$K = 5$' curve, at any given value of f_{c}, can be interpreted as a **robustness premium** obtained by investing in more massive stiffness. For instance, at $f_{\mathrm{c}} = 0.003$, changing from the '$K = 4$' to the '$K = 5$' design causes $\widehat{\alpha}$ to increase from 0.287 to 0.296, which constitutes a robustness premium of 0.009. This is about a 3% improvement in robustness.

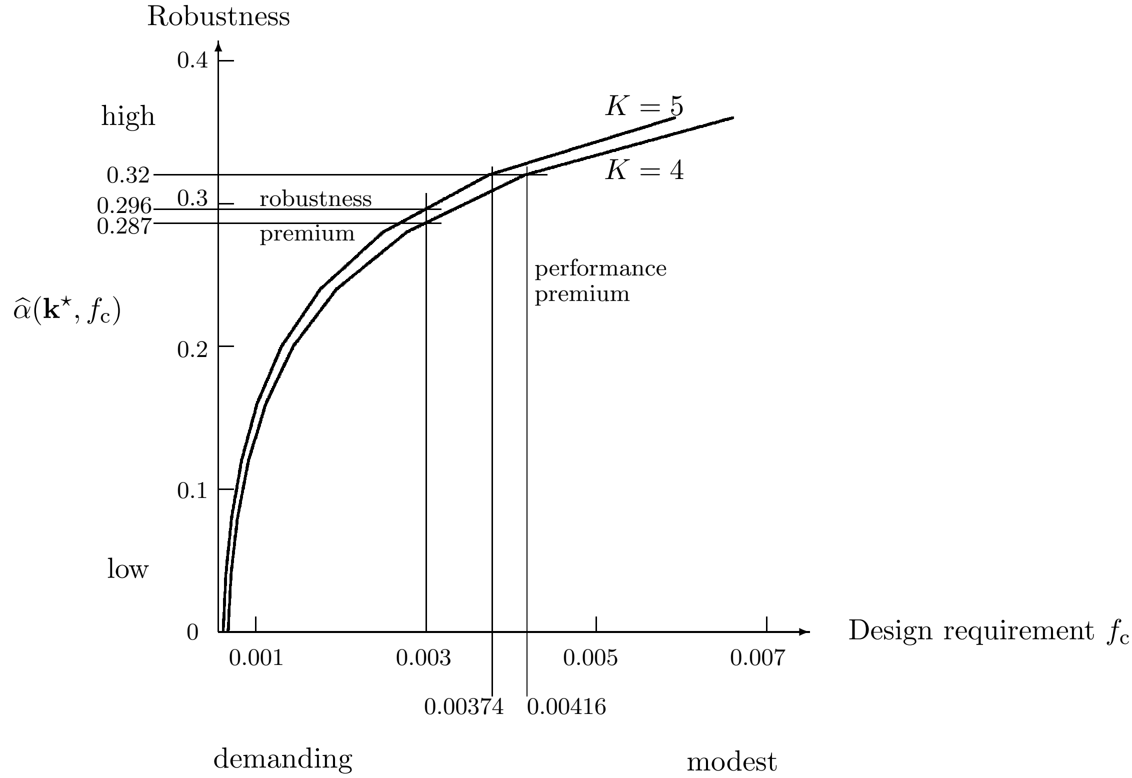

Figure 4: Robustness functions for two values of total stiffness: $K = 4$ and $K = 5$. In each case, $k_1^{\star} = 3k_2^{\star}/2$.

The same idea can be expressed in an alternative manner. The '$K = 5$' curve lies to the left of the '$K = 4$' curve in fig. 4, which means that more massive stiffness entails better guaranteed performance, at the same level of robustness to uncertainty. The horizontal distance from the '$K = 4$' curve to the '$K = 5$' curve, at any given value of $\widehat{\alpha}$, can be interpreted as a **performance premium** obtained by investing in more massive stiffness. For instance, at robustness of $\widehat{\alpha} = 0.32$, the performance premium is $0.00416 - 0.00374 = 0.00042$. This is about a 10% reduction in the sum of the interstory drift variances.

Robustness premia and related ideas are discussed in [2].

6 Relation between Robust-satisficing Optimal Design and Performance Optimal Design

The info-gap robust-satisficing optimal design defined in eq.(2) is very different, conceptually and practically, from performance-optimization of design, as we have seen in the example in section 5.2. We will now more thoroughly explain the difference, and why the robust-satisficing approach is preferable if there is severe uncertainty.

The usual approach to design optimization is based on a best-estimate of the dynamic model, $\widetilde{F}(\omega, \delta)$, where δ is the vector of design parameters. In optimizing the performance, one seeks the design which minimizes the performance function, $\widehat{f}(\widetilde{F}, \delta)$, based on this best dynamic model. The **performance-optimal design** is the value of δ which minimizes $\widehat{f}(\widetilde{F}, \delta)$:

$$\delta^{\star} = \arg\min_{\delta} \widehat{f}(\widetilde{F}, \delta) \tag{28}$$

To understand why the performance-optimal design, eq.(28), is very different from the info-gap robust-satisficing optimal design, eq.(2), we need to identify the level of performance at which the

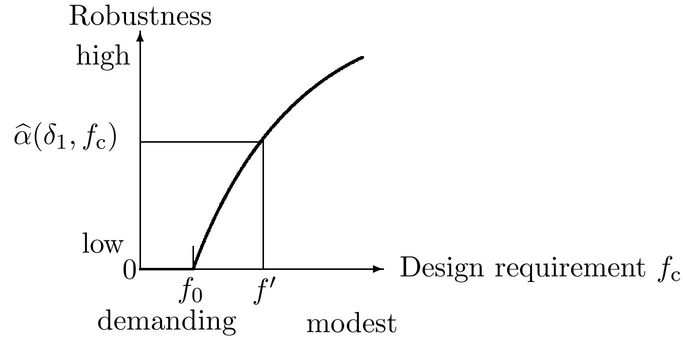

Figure 5: Illustration of the trade-off between robustness and performance.

robustness function reaches zero. Referring to the trade-off between robustness-to-uncertainty and performance which is illustrated schematically in fig. 5, we need to know the value of f_0 at which the robustness becomes zero.

The answer comes from a basic theorem in info-gap decision theory which, in the present context, asserts [4, section 11.6]:

$$\widehat{\alpha}(\delta, f_c) = 0 \quad \text{if} \quad f_c = \widehat{f}(\widetilde{F}, \delta) \tag{29}$$

(We have seen specific realizations of this in eqs.(11) and (20).) $\widehat{f}(\widetilde{F}, \delta)$ is the level of performance expected from design δ, based on the best-estimate of the dynamic behavior, $\widetilde{F}$. $\widehat{\alpha}(\delta, fc)$ is the robustness to uncertainty in the model, based on an info-gap model in which $\widetilde{F}$ is the centerpoint. Eq.(29) states that the robustness-to-uncertainty is zero at the level of performance predicted by the best-estimated dynamic model. What this means is that **no design can be depended upon to perform at the level which is predicted by the design-base dynamic model.**

In fact, we have already recognized the practical significance of this result. Recalling our discussion of fig. 3, we noted that performance $\gamma\widetilde{c}/m$ cannot be relied upon to occur. In the notation of fig. 5, performance as good as f_0 cannot be relied upon to occur with design δ_1. However, some poorer level of performance, $f' > f_0$, does have large and dependable robustness against uncertainty. That is, design δ_1 can be used if f' is an acceptable performance.

An important special case is of particular interest. Eq.(29) is true for *any* design, δ, so it is true for the performance-optimal design, $\delta^\star$ defined in eq.(28). That is:

$$\widehat{\alpha}(\delta^\star, f^\star) = 0 \quad \text{if} \quad f^\star = \widehat{f}(\widetilde{F}, \delta^\star) \tag{30}$$

This means that the performance-optimal design, $\delta^\star$, cannot be relied upon to perform as well as expected from the best-estimate of the dynamic model. Nonetheless, $\delta^\star$ can be relied upon to perform as well as some poorer level of performance, $f' > f_0$. However, there may be some other design, sub-optimal in performance, which also satisfices the performance at f', and which is more robust than $\delta^\star$. This is illustrated here in fig. 6. We have encountered this in fig. 2.

The optimality of design $\delta^\star$ is expressed in the fact that its robustness curve hits the horizontal axis further to the left than the robustness curve of any other design, meaning that the critical performance level, $f^\star$, is better than (less than) all other attainable values. However, the robustness-to-uncertainty of obtaining $f^\star$ is zero. Design δ_2 is more robust to uncertainty than the performance-optimal design $\delta^\star$, at performance requirement f'. Hence design δ_2 is preferable to design $\delta^\star$.

7 Load and Model Uncertainties: Generalization

We have concentrated on the designer's imperfect knowledge of the dynamic model of the structure. This model-uncertainty has been represented with the info-gap model $\mathcal{F}(\alpha, \widetilde{F})$, $\alpha \geq 0$. The load-uncertainty has been treated differently, by assuming that we know the maximum, $\overline{s}$, of the spectral density $S(\omega)$, and then considering the worst-case critical spectral density as described in [14, 15].

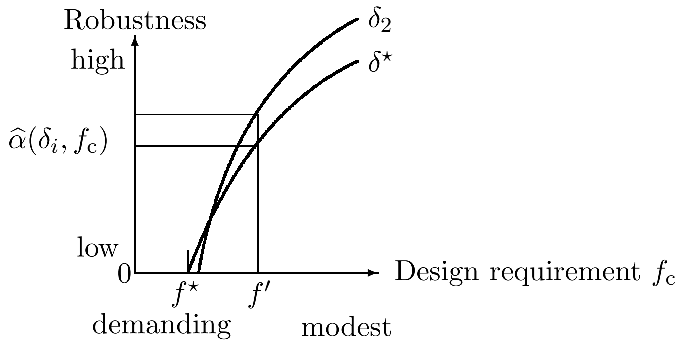

Figure 6: Illustration of the preference for sub-optimal design δ_2 over performance-optimal $\delta^\star$, at performance level f'.

In practice we do not know the maximum of the spectral density, and it is unrealistic to assume that $S(\omega)$ will necessarily adopt its critical form. In fact, when $\overline{s}$ is unknown, the set of possible spectral densities is unbounded and there is no worst case. Unbounded uncertainty of this type is particularly amenable to info-gap analysis. In this section we will formulate the info-gap robust-satisficing design procedure applied to both load and model uncertainties. An example is discussed in section 8.

7.1 Robustness function

The info-gap model for uncertainty in the dynamic model is $\mathcal{F}(\alpha_{\rm m}, \widetilde{F})$, $\alpha_{\rm m} \geq 0$. A different info-gap model is used to represent uncertainty in the spectral density: $\mathcal{S}(\alpha_{\rm s}, \widetilde{S})$, $\alpha_{\rm s} \geq 0$, where $\widetilde{S}$ is the nominal design-base spectral density. Note that we use two different horizon-of-uncertainty parameters: $\alpha_{\rm m}$ for model-uncertainty and $\alpha_{\rm s}$ for uncertainty in the spectral density of the load.

$\widehat{f}(S, F, \delta)$ is the vector of system responses to load spectrum S, based on model F and design δ. As in relation (6), the performance requirements are:

$$\widehat{f}_i(S, F, \delta) \leq f_{{\rm c},i}, \quad i = 1, \ldots, R \tag{31}$$

The robustness question is: how much can the nominal design-base dynamic model $\widetilde{F}$ err, as a function of uncertainty in the spectral density S, without jeopardizing the performance of the system? The answer to this question is expressed by the robustness function which is, in analogy to eq.(7):

$$\widehat{\alpha}_{\rm m}(\delta, f_{\rm c}, \alpha_{\rm s}) = \max \left\{ \alpha_{\rm m} : \left[\max_{\substack{F \in \mathcal{F}(\alpha_{\rm m}, \widetilde{F}) \\ S \in \mathcal{S}(\alpha_{\rm s}, \widetilde{S})}} \widehat{f}_i(S, F, \delta) \right] \leq f_{{\rm c},i}, \text{ for all } i = 1, \ldots, R \right\} \tag{32}$$

$\widehat{\alpha}_{\rm m}(\delta, f_{\rm c}, \alpha_{\rm s})$ is the robustness to model-uncertainty, as a function of the horizon of uncertainty $\alpha_{\rm s}$ in the spectral density. We can "read" eq.(32) from left to right as follows. For given design δ, performance requirements $f_{\rm c}$ and load-uncertainty $\alpha_{\rm s}$, the robustness to model-uncertainty, $\widehat{\alpha}_{\rm m}(\delta, f_{\rm c}, \alpha_{\rm s})$, is the maximum value of $\alpha_{\rm m}$ such that all models F up to horizon of model-uncertainty $\alpha_{\rm m}$, and all spectral densities S up to horizon of load-uncertainty $\alpha_{\rm s}$, result in acceptable responses $\widehat{f}_i(S, F, \delta)$ for all $i = 1, \ldots, R$.

7.2 Info-gap Models of Spectral Uncertainty

Several info-gap models are available for representing uncertainty in the spectral density of the load. The simplest model is based on using the rectangular critical excitation, which consists of two rectangular spectra located symmetrically around the origin on the frequency axis. Let $\widetilde{S}(\omega; \Delta\omega, \overline{S})$ denote the rectangular density whose total area (of both rectangles) is $\overline{S}$ and for which the width of each rectangle is $\Delta\omega$. We consider uncertainty only in the height and width of these rectangular spectral densities with the following info-gap model:

$$\mathcal{S}[\alpha_{\rm s}, \widetilde{S}(\omega; \Delta\omega, \overline{S})] = \left\{ S(\omega) = s^* \widetilde{S}(\omega; \Delta\omega/s^*, \overline{S}) : \; s^* = s/\widetilde{s}, \; \left| \frac{s - \widetilde{s}}{\widetilde{s}} \right| \le \alpha_{\rm s} \right\}, \quad \alpha_{\rm s} \ge 0 \qquad (33)$$

Note that the rectangles of $s^* \widetilde{S}(\omega; \Delta\omega/s^*, \overline{S})$ are just as wide as, but s^* times higher than, the rectangles of $\widetilde{S}(\omega; \Delta\omega/s^*, \overline{S})$. $\widetilde{s}$ is the nominal value of the load spectral density.

In other situations one may wish to use a specific spectral form, such as the Kanai-Tajimi spectral density [13] or a site-specific historical spectral density. Let $\widetilde{S}(\omega)$ denote the known, nominal, design-base spectral density of choice. The **interval-bound info-gap model** for uncertainty in the actual spectral density is the unbounded family of nested sets of densities which deviate from $\widetilde{S}(\omega)$ within an envelope of known shape and unknown size:

$$\mathcal{S}(\alpha_{\rm s}, \widetilde{S}) = \left\{ S(\omega) : \; \left| S(\omega) - \widetilde{S}(\omega) \right| \le \alpha_{\rm s} \psi(\omega) \right\}, \quad \alpha_{\rm s} \ge 0 \qquad (34)$$

where $\psi(\omega)$ is the known envelope function and $\alpha_{\rm s}$ is the unknown horizon of uncertainty.

Alternatively, the **energy-bound info-gap model** is the unbounded family of nested sets of spectral densities whose integral squared-deviation from the nominal density is bounded, but where the value of this bound, $\alpha_{\rm s}$, is unknown:

$$\mathcal{S}(\alpha_{\rm s}, \widetilde{S}) = \left\{ S(\omega) : \; \int_0^\infty [S(\omega) - \widetilde{S}(\omega)]^2 \, {\rm d}\omega \le \alpha_{\rm s}^2 \right\}, \quad \alpha_{\rm s} \ge 0 \qquad (35)$$

8 Example: 6-story Shear Building with Load and Model Uncertainties

In this section we illustrate the info-gap robust-satisficing design of a 6-story shear building subjected to stationary random ground acceleration. The spectral density of the ground motion, and the damping coefficients of the linear dynamic model of the building vibration, are all uncertain.

8.1 Formulation

The performance function is the sum of the variances of the inter-story drifts, $\widehat{f}(\mathbf{c}, \mathbf{k})$, defined in eq.(25) and adapted to the 6-story case. $\mathbf{c}$ and $\mathbf{k}$ are the vectors of damping and stiffness coefficients.

The known, nominal, damping coefficients are $\widetilde{c}_i = 5.41 \times 10^5$ N·s/m, $i = 1, \ldots, 6$. The uncertainty in the damping coefficients is represented by the info-gap model of eq.(26). The known nominal spectral density is the rectangular critical excitation with area $\overline{S} = 0.553$ m^2/s^4. The width of each nominal spectral rectangle is $\Delta\omega = 4.189$ rad/s. The uncertainty in the amplitude of the rectangular critical spectral density is represented by the info-gap model of eq.(33). Different horizon-of-uncertainty parameters are used for model and load uncertainties, $\alpha_{\rm m}$ and $\alpha_{\rm s}$ respectively. However, they both have the meaning of fractional deviation from the nominal values. The mass of each of the 6 stories is $m_i = 32 \times 10^3$ kg.

We will analyse the robustness to load and model uncertainties for two choices of the shear stiffness coefficients $\mathbf{k}$. In both designs the sum of the stiffnesses is $K = 0.319 \times 10^9$ N/m.

In the 'FC design' [15] the fundamental natural period is 0.60 s, and the normalized stiffness coefficients are related by: $k_i = \sum_{j=i}^{6} j$, where k_i is the shear stiffness of the ith floor. These stiffnesses, multiplied by a unit interstory drift, provide shear forces which precisely balance the inertial forces in the fundamental vibrational mode, whose deflection increases linearly from the ground.

In the 'constant-$\mathbf{k}$ design' the normalized stiffnesses are all equal: $k_i = K/6$, $i = 1, \ldots, 6$. The fundamental natural period is 0.64 s.

8.2 Trade-off Between Performance and Robustness to Uncertainty

The robustness $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$ of design $\mathbf{k}$ is defined in eq.(32), given the performance requirement that the sum of the drift-variances $\widehat{f}(\mathbf{c}, \mathbf{k})$ not exceed the critical value $f_{\rm c}$, and conditioned on info-gap uncertainty $\alpha_{\rm s}$ in the height of the spectral density.

The evaluation of the inner maxima in eq.(32), up to horizons of uncertainty $\alpha_{\rm m}$ and $\alpha_{\rm s}$, is straightforward. The inner maximum on the model-uncertainty occurs when each damping coefficient errs maximally as $c_i = \widetilde{c}_i(1 - \alpha_{\rm m})$. The inner maximum on the spectral-uncertainty occurs when the spectral amplitude errs maximally as $s = \widetilde{s}(1 + \alpha_{\rm s})$.

Two trade-offs characterize the robustness to model-uncertainty, $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$. As before, the robustness decreases as the performance improves: $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$ gets smaller as the performance requirement $f_{\rm c}$ decreases. Furthermore, the robustness to model-uncertainty decreases as the load-uncertainty increases: $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$ gets smaller as $\alpha_{\rm s}$ gets larger. These trade-offs are illustrated in figs. 7 and 8 which we now discuss.

Robustness curves for both the FC design and the constant-**k** design are shown in fig. 7 for an info-gap in the spectral density equal to $\alpha_{\rm s} = 0.3$. Fig. 7 shows that the FC design is slightly more robust than the constant-**k** design at all levels of demanded performance $f_{\rm c}$. (This is true at all values of $\alpha_{\rm s}$ which were examined.) While the curves are close, nonetheless the performance premium of the FC design over the constant-**k** design is not negligible. For instance, at 30% robustness ($\widehat{\alpha}_{\rm m} = 0.3$), the performance guaranteed by the FC design and the constant-**k** design are, respectively, $f_{\rm c} = 0.00658$ and $f_{\rm c} = 0.00755$. The performance premium of the FC design is about 13%. It is important to stress that the comparison of these two designs is made at a positive value of robustness. Comparison on the $\widehat{\alpha}_{\rm m} = 0$ axis would be misleading: these two designs have very nearly the same performance according to the nominal design-base data (they intersect the $f_{\rm c}$-axis nearly together). However, the robustness-to-uncertainty of this nominal performance is zero.

Fig. 8 shows the trade-off between robustness to model uncertainty, $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$, and info-gap uncertainty $\alpha_{\rm s}$ in the amplitude of the spectral density. The three pairs of curves are for three different values of the demanded performance $f_{\rm c}$. In each pair of curves, the FC and the constant-**k** designs are indicated by the '$\star$' and the '$\circ$' respectively. These curves all have negative slope, indicating that as the load-uncertainty increases, the structure becomes less immune to uncertainty in the dynamic model.

The slopes of the curves in fig. 8 typically equal about -0.06. This means that an increase in spectral uncertainty of 10% ($\Delta\alpha_{\rm s} = +0.1$) results in a decrease in robustness to model uncertainty of about 0.6% ($\Delta\widehat{\alpha}_{\rm m} = -0.006$). The **model-robustness cost** of imperfect knowledge of the load is quite low.

We have explained that $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$ is the greatest horizon of model-uncertainty which is tolerable, subject to performance requirement $f_{\rm c}$, given a spectral info-gap equal to $\alpha_{\rm s}$. It is just as correct to invert the interpretation: $\alpha_{\rm s}$ as the greatest tolerable horizon of spectral uncertainty, subject to performance requirement $f_{\rm c}$, given an info-gap in the model equal to $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$. This conclusion results from the trade-off between knowledge of the model and knowledge of the load. This trade-off is a universal property, and is manifested in the negative slopes of the curves in fig. 8. If the spectral uncertainty exceeds $\alpha_{\rm s}$, then performance as good as $f_{\rm c}$ cannot be guaranteed unless the model-uncertainty is reduced below $\widehat{\alpha}_{\rm m}(\mathbf{k}, f_{\rm c}, \alpha_{\rm s})$. We can now interpret the slightly negative slope of the curves in fig. 8 to indicate a very high **load-robustness cost.** Given a slope of the curves in fig. 8 equal to -0.06, an increase in model-uncertainty of only 1% ($\Delta\widehat{\alpha}_{\rm m} = +0.01$) results in a loss of load-robustness of about 17% ($\Delta\alpha_{\rm s} = -0.17$).

9 Conclusion

This paper has developed a new methodology for design of civil structures subject to severe uncertainties in both the loads and the models upon which the design is based. The following are the main ideas of the paper.

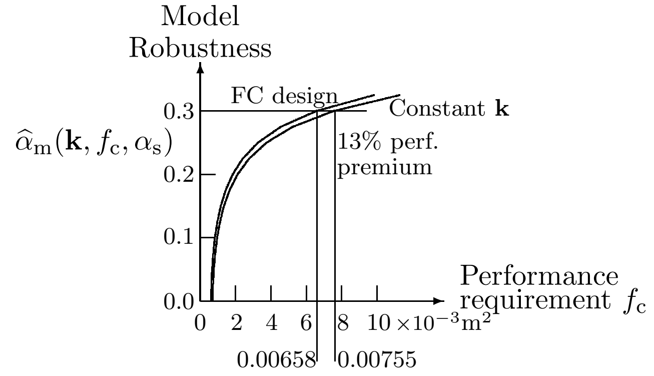

Figure 7: Model-robustness vs. performance requirement for both stiffness designs. Curves of constant load-uncertainty $\alpha_s = 0.3$.

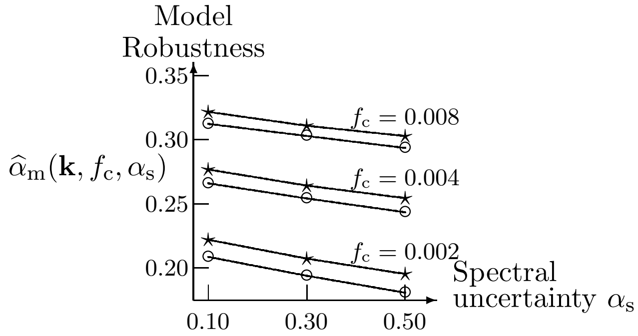

Figure 8: Model-robustness vs. spectral uncertainty. Curves of constant performance requirement f_c. FC design ($\star$), constant **k** design ($\circ$)

1. Critical excitations depend upon dynamic properties of the structure. Consequently it is necessary to treat load and model uncertainties together. In this paper, info-gap models of uncertainty are used to describe uncertainty in the upper bound of the PSD of the load and in the parameters of the vibration model of the structure. Our illustrative examples employ simple info-gap models. More information-intensive info-gap models are also discussed.

2. Any design which optimizes the functional performance of a structure will also minimize its robustness to uncertainty. That is, there is an unavoidable trade-off between the functional performance of a structure and its robustness to uncertainties in the knowledge upon which the design is based. This trade-off occurs because the knowledge base is "strained" to accomodate demanding performance, so small errors can cause performance shortfall.

3. Because of this trade-off and because uncertainties are dominant in many applications, we have argued that it is necessary to **satisfy critical performance requirements** (rather than to optimize performance), and to **maximize the robustness to uncertainty.** We have referred to this design methodology as **robust-satisficing.**

4. We have illustrated these theoretical conclusions with several heuristic design examples. The need for robustness-to-uncertainty induces the designer to satisfice rather than optimize structural performance. This can result in design preferences which depend on performance requirements. Fig. 2 shows that one design is preferred when low performance is adequate, while another design is preferred at more demanding performance requirements. This **preference reversal** occurs when the robustness curves cross. The need for positive robustness can also cause the performance-optimal design to be less preferred than the optimal robust-satisficing design, at any level of performance at which the robustness is positive, as illustrated in fig. 3. The concepts of **robustness premium** and **performance premium** are illustrated in fig. 4, which compares a lighter and a more massive design.

5. The simultaneous consideration of load and model uncertainties introduces a new trade-off, illustrated in fig. 8. The robustness to model uncertainty increases as the horizon of load uncertainty gets smaller. This leads to the idea of information costs: **load-robustness cost** and **model-robustness cost.**

6. We have shown that the info-gap robustness function, eq.(7), provides an attractive tool for adjudicating between conflicting objectives in multi-criteria design. This arises because the robust-satisficing design strategy entails satisfactory but sub-optimal performance for all criteria, and optimization of the overall robustness.

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