27. **Uncertain lotteries.** (p.166) Consider a lottery with two prizes whose values are $v_\ell > v_\mathrm{s}$. Each participant wins either the large prize or the small prize. The probability of winning the larger prize is uncertain; the best estimate of this probability if $\widetilde{p}$; and the info-gap model for uncertainty in the probability is:

$$\mathcal{U}(h, \widetilde{p}) = \left\{ p : \ 0 \le p \le 1, \ \left| \frac{p - \widetilde{p}}{\widetilde{p}} \right| \le h \right\}, \quad h \ge 0 \qquad (90)$$

(a) For any critical value of the expected reward v_c, such as the cost of a lottery ticket, what is the robustness, to uncertainty in $\widetilde{p}$, of winning at least v_c on average?

(b) Now consider a different lottery with prizes $v'_\ell > v'_\mathrm{s}$ and estimated probability $\widetilde{p}'$ of winning v'_ℓ. Furthermore, the estimated average prize is now greater: $\widetilde{p}' v'_\ell + (1 - \widetilde{p}') v'_\mathrm{s} > \widetilde{p} v_\ell + (1 - \widetilde{p}) v_\mathrm{s}$. However, the smaller prize is now even smaller: $v'_\mathrm{s} < v_\mathrm{s}$. The uncertainty of the probability is represented with the info-gap model of eq.(90), now centered on $\widetilde{p}'$. Under what conditions (e.g., with what values of v_c) will you prefer this new lottery? (Consider the crossing of the robustness curves of these two lotteries.)