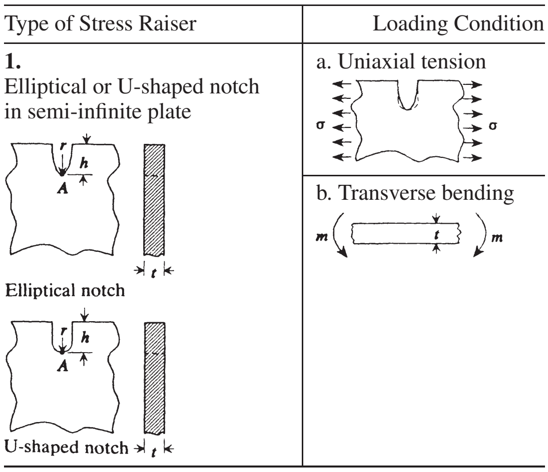

Figure 18: Stress concentration geometry for problem 80.

80. **Stress concentration factor** (035018, based on exam, 31.5.2015), (p.271). Consider a small notch in the surface of a large solid under uniaxial tension σ, as in fig. 18.[8] The depth of the notch is d (denoted h in the figure) and the radius of curvature of the tip of the notch is r. The stress concentration factor (SCF), K, is the ratio of the maximal stress at the tip of the notch to the stress, σ, far from the notch. A theoretically based empirical relation is:

$$K = a + b\sqrt{\frac{d}{r}} \tag{356}$$

where a and b are positive empirical coefficients.

(a) The radius of curvature is estimated to be $\tilde{r}$ with error s_r. The fractional error is unknown and uncertainty is represented with this info-gap model:

$$\mathcal{U}(h) = \left\{ r: \ r \geq 0, \ \left| \frac{r - \tilde{r}}{s_r} \right| \leq h \right\}, \quad h \geq 0 \tag{357}$$

We require that the SCF be no greater than the critical value K_{c}:

$$K \leq K_{\mathrm{c}} \tag{358}$$

Derive an explicit algebraic expression for the robustness.

(b) Define $c = (a, b)^T$ as the vector of coefficients in eq.(356) and define the vector $g = (1, \sqrt{d/r})^T$. Thus $K = c^T g$. The coefficients are uncertain as represented by an ellipsoid-bound info-gap model:

$$\mathcal{U}(h) = \left\{ c: \ (c - \tilde{c})^T W (c - \tilde{c}) \leq h^2 \right\}, \quad h \geq 0 \tag{359}$$

where $\tilde{c}$ is a known vector and W is a known, real, positive definite, symmetric matrix. The performance requirement is eq.(358). Derive an explicit algebraic expression for the robustness.

(c) Assume that the radius, r, in eq.(356) is a random variable with a normal distribution with mean μ and variance s^2. The probability of failure, which we denote P_{f}, is the probability of violating eq.(358). Derive an explicit algebraic expression for P_{f}.

[8] This figure is from http://www.ewp.rpi.edu/hartford/~ernesto/Su2012/EP/MaterialsforStudents/Aiello/Roark-Ch06.pdf

(d) Continuing part 80c, let the mean, μ, and standard deviation, s, be uncertain according to this fractional-error info-gap model:

$$\mathcal{U}(h) = \left\{ \mu, s : \left| \frac{\mu - \widetilde{\mu}}{\widetilde{\mu}} \right| \le h, \ s \ge 0, \ \left| \frac{s - \widetilde{s}}{\widetilde{s}} \right| \le h \right\}, \quad h \ge 0 \tag{360}$$

We require that the probability of failure be no greater than the critical value $P_{\rm c}$:

$$P_{\rm f} \le P_{\rm c} \tag{361}$$

Combining eqs.(356) and (358), let us define a "critical radius" in terms of the critical SCF: $r_{\rm c} = d\left(\frac{b}{K_{\rm c}-a}\right)^2$. Assume that the estimated mean, $\widetilde{\mu}$, exceeds $r_{\rm c}$. Derive an explicit algebraic expression for the inverse of the robustness function.

(e) Consider the SCF at small radii, for which eq.(356) can be approximated as:

$$K = b\sqrt{\frac{d}{r}} \tag{362}$$

The estimated values of the coefficient b for two different materials are $\widetilde{b}_1$ and $\widetilde{b}_2$ where $\widetilde{b}_1 > \widetilde{b}_2$. However, the fractional errors of the true values are uncertain, as represented by this info-gap model:

$$\mathcal{U}(h) = \left\{ b_1, b_2 : \ b_i \ge 0, \ \left| \frac{b_i - \widetilde{b}_i}{\widetilde{b}_i} \right| \le h, \ i = 1, 2 \right\}, \quad h \ge 0 \tag{363}$$

For each material, derive an explicit algebraic expression for the robustness of satisfying the performance requirement in eq.(358). For what range of $K_{\rm c}$ values do you robustly prefer option 2?

(f) Consider the SCF at all radii, for which we must use eq.(356). The estimated values of the coefficients a and b for two different materials are $(\widetilde{a}_1, \widetilde{b}_1)$ and $(\widetilde{a}_2, \widetilde{b}_2)$ where $\widetilde{a}_1 < \widetilde{a}_2$ and $\widetilde{b}_1 > \widetilde{b}_2$. Let $K_i(r)$ denote the SCF for material i as a function of the radius r. From eq.(356) we see that $K_1(r) > K_2(r)$ at small radii, and $K_1(r) < K_2(r)$ at large radii. Let $r_\times$ denote the radius at which the SCF curves of the two materials cross.

The fractional errors of the true values of a_i and b_i are uncertain, as represented by this info-gap model:

$$\mathcal{U}(h) = \left\{ (\widetilde{a}_i, \widetilde{b}_i) : \ a_i \ge 0, \ \left| \frac{a_i - \widetilde{a}_i}{\widetilde{a}_i} \right| \le h, \ b_i \ge 0, \ \left| \frac{b_i - \widetilde{b}_i}{\widetilde{b}_i} \right| \le h, \ i = 1, 2 \right\}, \quad h \ge 0 \tag{364}$$

For each material, derive an explicit algebraic expression for the robustness of satisfying the performance requirement in eq.(358). For what range of $K_{\rm c}$ values do you robustly prefer material 1?