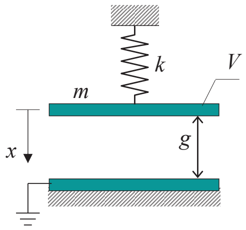

Figure 14: Gap-closing electrostatic actuator for problem 72. (Fig. thanks to Prof. David Elata, head, Mechanical Engineering Micro Systems (MEMS) lab, Technion.)

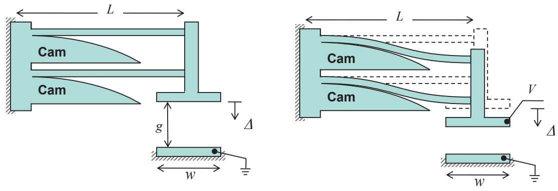

Figure 15: Mechanically linearized Gap-closing electrostatic actuator for problem 72. (Fig. thanks to Prof. David Elata)

72. **Gap-closing electrostatic actuators,** (p.250). The non-linear force-displacement relation for the gap-closing electrostatic actuator in fig. 14 is:

$$F = kx - \frac{\varepsilon A V^2}{2(g-x)^2} \tag{285}$$

where ε is the dielectric constant, A is the area of the plates, V is the electric potential on the device, k is the spring stiffness and g is the initial gap size.

Fig. 15 shows a mechanically linearized modification of the device in fig. 14 for which the force-displacement relation is, nominally, linear:

$$F = Kx \tag{286}$$

The degree of linearity depends on the shapes of the cams and on the degree of mechanical and structural uniformity of the pair of beams. We will explore the robustness to various forms of uncertainty in the linearity of the beam. We will also explore probabilistic models and statistical decisions.

(a) We require that application of a known force F results in a displacement no less than x_{c}. Uncertainty in the linear stiffness coefficient K is represented by the info-gap model:

$$\mathcal{U}(h) = \left\{ K : \ K > 0, \ \left| \frac{K - \widetilde{K}}{s} \right| \le h \right\}, \quad h \ge 0 \tag{287}$$

where $\widetilde{K}$ is the known nominal linear stiffness and s is a known positive error coefficient. Derive an explicit algebraic expression for the robustness.

(b) Repeat part 72a where we aspire to displacement as large as x_{w}. Derive an explicit algebraic expression for the opportuneness.

(c) We require that application of a known force F results in a displacement no less than x_{c}. However, the nominal linear force-displacement relation in eq.(286) is replaced by:

$$x = \frac{F}{K} + \sum_{n=1}^{N} a_n F^n = \frac{F}{K} + a^T \phi \tag{288}$$

where ϕ is the vector of powers of F and a is the vector of coefficients whose uncertainty is represented by the info-gap model:

$$\mathcal{U}(h) = \left\{ a : \ a^T W a \le h^2 \right\}, \quad h \ge 0 \tag{289}$$

where W is a known, real, symmetric, positive definite matrix. Derive an explicit algebraic expression for the robustness.

(d) Now consider K in eq.(286) to be a random variable with a uniform probability density:

$$p(K) = \frac{1}{K_{\max}}, \quad 0 \le K \le K_{\max} \tag{290}$$

Failure occurs if

$$x < x_{\rm c} \tag{291}$$

Derive an explicit algebraic expression for the probability of failure. Assume $F \le x_{\rm c} K_{\max}$.

(e) Continuing part 72d, suppose that F and $K_{\max}$ are both info-gap-uncertain as described by the following info-gap model:

$$\mathcal{U}(h) = \left\{ F, K_{\max} : \ F > 0, \ \left| \frac{F - \widetilde{F}}{\widetilde{F}} \right| \le h, \ K_{\max} > 0, \ \left| \frac{K_{\max} - \widetilde{K}_{\max}}{\widetilde{K}_{\max}} \right| \le h, \right\}, \quad h \ge 0 \tag{292}$$

We require that the probability of failure not exceed the critical value $P_{\rm c}$. Derive an explicit algebraic expression for the robustness. Assume $\widetilde{F} \le x_{\rm c} \widetilde{K}_{\max}$.

(f) Let K be a random variable whose estimated pdf, $\widetilde{p}(K)$, is normal with mean μ and variance σ^2. We are confident that this estimate is accurate for K within an interval around μ of known size $\pm\delta_s$. However, outside of this interval of K values the fractional error of the pdf is unknown. Our info-gap model is:

$$\mathcal{U}(h) = \left\{ p(K) : \quad \begin{array}{l} \int_{-\infty}^{\infty} p(K)\,{\rm d}K = 1, \ p(K) \ge 0, \ \forall K, \\ p(K) = \widetilde{p}(K), \ |K - \mu| \le \delta_s \\ \left| \dfrac{p(K) - \widetilde{p}(K)}{\widetilde{p}(K)} \right| \le h, \ |K - \mu| > \delta_s \end{array} \right\}, \ h \ge 0 \tag{293}$$

The system fails if $x < x_{\rm c}$ where $x = F/K$ as stated in eq.(286), where F is a known positive constant. x is now a random variable so the performance requirement is that the probability of failure not exceed a critical value $P_{\rm c}$. Derive an explicit algebraic expression for the robustness function. Assume that $F/x_{\rm c} \ge \mu + \delta_s$.

(g) We are testing a MEMS system but we don't know if it is "raw" like fig. 14 or "linearized" like fig. 15. The loads are random and, if the beam is linearized, they produce small, medium and large deflections with frequencies 0.5, 0.3 and 0.2, respectively. If the beam is not linearized then the frequencies are different. We observe 41 small, 32 medium, and 27 large displacements. For the following hypotheses, do you accept or reject H_0 at the 0.05 level of significance?

$$H_0 : \quad p_{\rm sml} = 0.5, \ \ p_{\rm med} = 0.3, \ \ p_{\rm lrg} = 0.2 \tag{294}$$

$$H_1 : \quad \neg H_0 \tag{295}$$

(h) The lifetime of the device is distributed according to a Weibull distribution whose probability distribution function is:

$$P(t) = 1 - {\rm e}^{-(\lambda t)^{\alpha}}, \quad t \ge 0 \tag{296}$$

where λ and α are positive constants. A specific unit has been observed to be operational at time t_0. Derive an explicit algebraic expression for the probability that this unit will be operational at time t_1.

(i) We have an endless supply of devices where a fraction p are "raw" like fig. 14 and the rest are "linearized" like fig. 15. We select N devices randomly and independently. Derive an explicit algebraic expression for the probability that J or more devices are "raw".

(j) Apply a known force, F, measure the resulting displacement x, and let y denote the difference between the measurement and the predicted displacement based on eq.(286). Assume the measurement is corrupted by zero-mean normally distributed noise whose variance is unknown. The values of y in a random sample of size $N = 6$ are 1, 3, 3, 4, 1, 2. The advocate of the linear model in eq.(286) claims that the true value of y is zero, while the critic claims that this is false:

$$H_0 : \qquad y = 0 \tag{297}$$

$$H_1 : \qquad y \neq 0 \tag{298}$$

Do you accept or reject H_0 at the 0.01 level of significance?