Info-Gap Theory
Concepts and Applications

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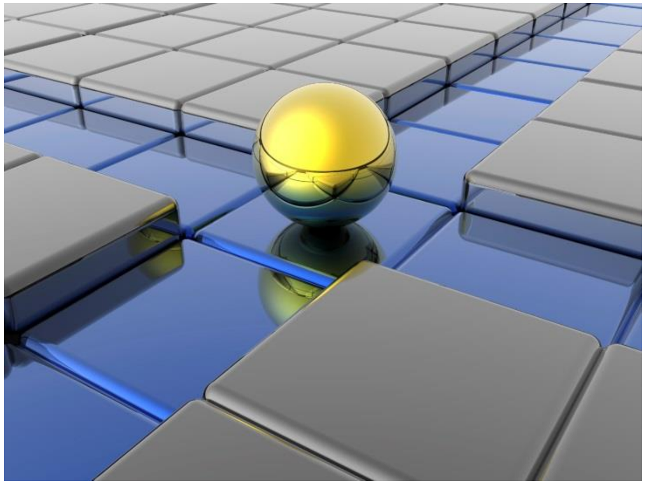


Outline

Risk or Uncertainty?

Probability is powerful, but ignorance is not probabilistic

Uncertainty and the optimization imperative

- Limits of prediction and outcome-optimization
- Robust satisficing

Time to Recovery: Innovation dilemma

Optimal monitoring and surveillance: A paradox

Risk and Uncertainty

Probabilistic risk

or

Knightian “true uncertainty”

Probabilistic Risk

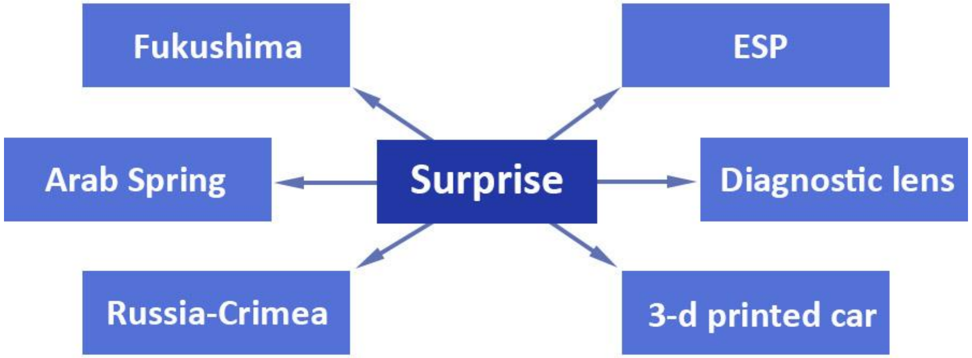
Consequence	Probability
Drought	Stochastic process
Industrial accident	Actuarial tables
Tsunami	Historical data
Faulty air filters	Quality control data
Deception, scam	Sociological data

Risk is:

- Structured: known event space
- Modeled with probability
- Manageable (but still risky)

Frank Knight's "true uncertainty"

"The uncertainties which persist … are **uninsurable** because there is **no objective measure of the probability**".



Wheeler's Island

"We live on an island of knowledge
surrounded by a sea of ignorance.
As our island of knowledge grows,
so does the shore of our ignorance."

John A. Wheeler

Non-probabilistic true uncertainty

D Discovery

- America
- Nuclear fission
- Martians (not yet?)

Non-probabilistic true uncertainty

D Discovery

I Invention/Innovation

- Printing press: material invention.
- Ecological responsibility: conceptual innovation.
- French revolution: social innovation.

Non-probabilistic true uncertainty

D Discovery

I Invention/Innovation

S Surprise (Asymmetric uncertainty)

- Ambush
- Competitor's innovation
- Natural catastrophe

Non-probabilistic true uncertainty

D Discovery

I Invention/Innovation

S Surprise (Asymmetric uncertainty)

What's the next D I or S ???

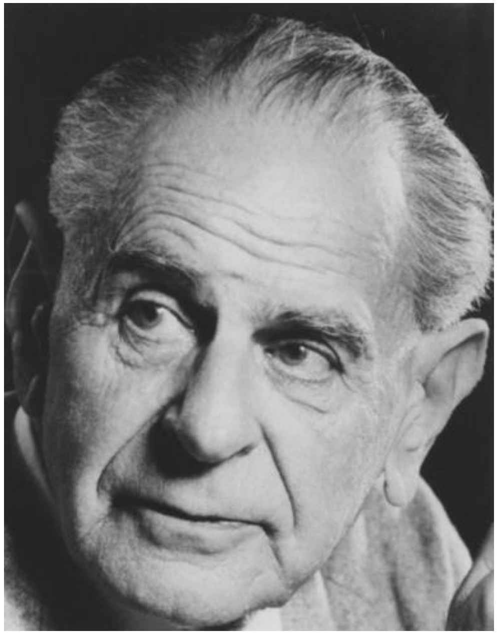
Knightian uncertainty:

- Unstructured: unknown event space.
- Indeterminate: no laws.
- Barely manageable.

Shackle-Popper

Indeterminism

GLS Shackle, 1903-1992



Karl Popper, 1902-1994

Shackle-Popper Indeterminism

Intelligence:

What people **know**, influences how they **behave**.

Discovery:

What will be **discovered tomorrow** can't be **known today**.

Implies

Indeterminism:

Tomorrow's behavior can't be fully modelled today.

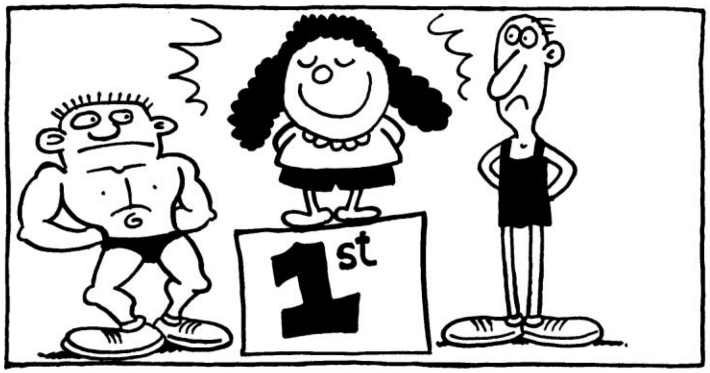
- Info-gaps, indeterminism: **unpredictable.**
- **Ignorance is not probabilistic.**

Uncertainty and the Optimization Imperative

Doing your best:

What does that mean?

- Outcome optimization.
- Procedural optimization.



Implications for decision making:

Robust satisficing.

Doing Your Best

Substantive outcome optimization:

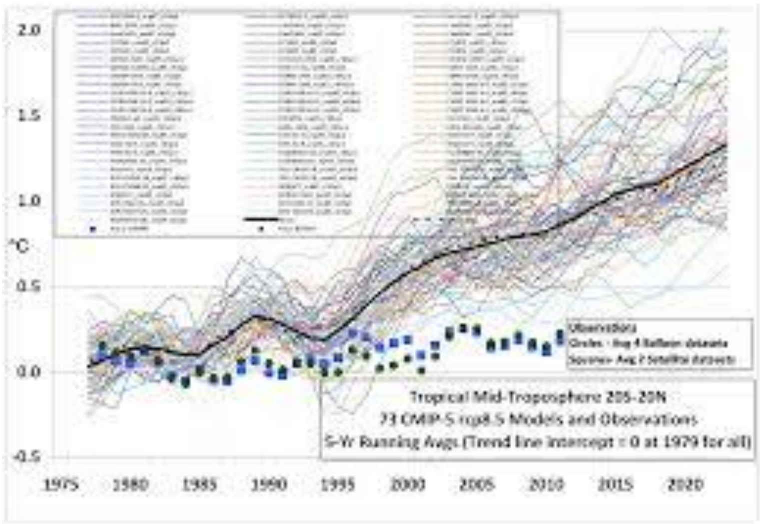
- Predict outcomes of available options.
- Select predicted best option.

Doing Your Best

Substantive outcome optimization.

Useful under risk:

- Structured uncertainty.
- Reliable probabilistic predictions.



Doing Your Best

Substantive outcome optimization:

Useful under risk.

Not useful (irresponsible?) under uncertainty.

- Unstructured uncertainty.
- Unreliable predictions.

Questions

What do we (not) know?

Robustness questions:

- What is an **essential** outcome?
- How to be **robust** to **surprise**?

Opportuneness questions:

- What is a **windfall** outcome?
- How to exploit **opportunities**?

How to prioritize decision options?

What are the trade offs?

Answers

Robustness answer:

System model
Outcome requirement
Uncertainty model

→ Robustness function → Prioritized options

Opportuneness answer:

System model
Outcome aspiration
Uncertainty model

→ Opportuneness function → Prioritized options

Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

1. Satisficing: Achieving critical outcomes.

- Essential goals.
- Worst acceptable outcomes.
- Modest or ambitious.

Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

1. Satisficing: Achieving critical outcomes.

2. Robustness:

- Immunity to ignorance.
- Greatest tolerable error or surprise.

Robust Satisficing

Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

1. Satisficing: Achieving critical outcomes.

2. Robustness: Greatest tolerable error.

Optimize robustness; satisfice goals:

Procedural (not substantive) **optimization.**

Time To Recovery

Time to recovery (TTR) after disruption:

- Building after earthquake.
- Energy distribution network after failure.
- Micro-sensor after shock load.
- Etc.

Task: Recover critical functions in specified time.

Challenge: Uncertainties (info-gaps).

Time To Recovery

Formulation: Innovation dilemma.

- **Choose between 2 design concepts:**
  - **State of the art (SotA, *q=0*).**
  - **New and innovative (NaI, *q=1*).**
- **System model: TTR, *t(a,q),* to load *a* for system *q*.**
- **Outcome requirement and aspiration:**

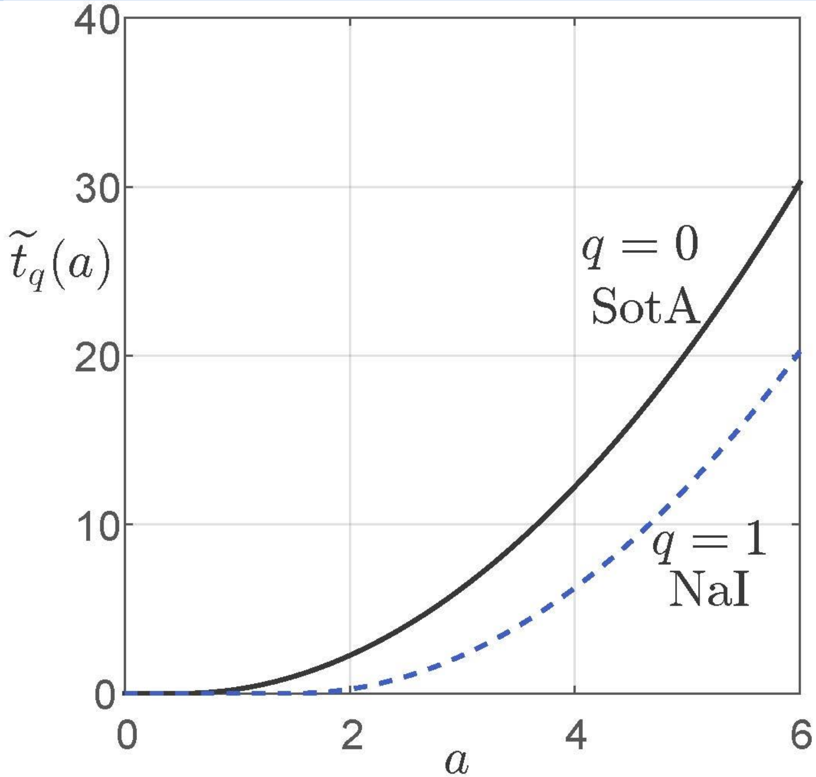
$$t(a,q) < tc, \qquad t(a,q) < tw \ (<< tc)$$

Info-gaps:

- **Parameter uncertainty: value of *a*.**
- **Functional uncertainty: shape of *t(a,q)*.**

Time To Recovery

Estimated TTR functions for 2 designs.



Putative preference: NaI predicted better than SotA.

What about uncertainty in load *a* & TTR func *t(a,q)*?

Time To Recovery

Info-gap:

Disparity between what we do know (on *a* & *t(a,q)*) and what we need to know in order to make responsible decision (SotA or NaI).

About the load, *a*:

Known estimated value. Unknown fractional error.

About the TTR function, *t(a,q)*:

- **Known estimated form. Unknown fractional error.**
- **NaI more uncertain than SotA.**

Time To Recovery

Info-gap model of uncertain *a* and *t(q,a)*:

$$\mathcal{U}(h) = \Big\{ a, t_q(a) : \ t_q(a) \ge 0, \ \ |t_q(a) - \widetilde{t}_q(a)| \le h w_q \widetilde{t}_q(a), \ q = 0, 1.$$

$$a > 0, \ \left| \frac{a - \widetilde{a}}{s} \right| \le h \Big\}, \quad h \ge 0 \qquad (18)$$

- **Non-prob: unbounded family of nested sets.**
- **Horizon of uncertainty, *h*, unknown.**
- **No known worst case.**
- **Axioms: Contraction and Nesting.**

Time To Recovery

Immunity functions.

Robustness: immunity against failure.

Maximum tolerable uncertainty.

$$\widehat{h}_q(t_c) = \max \left\{ h : \ \left(\max_{t,a \in \mathcal{U}(h)} t_q \right) \leq t_c \right\} \qquad (21)$$

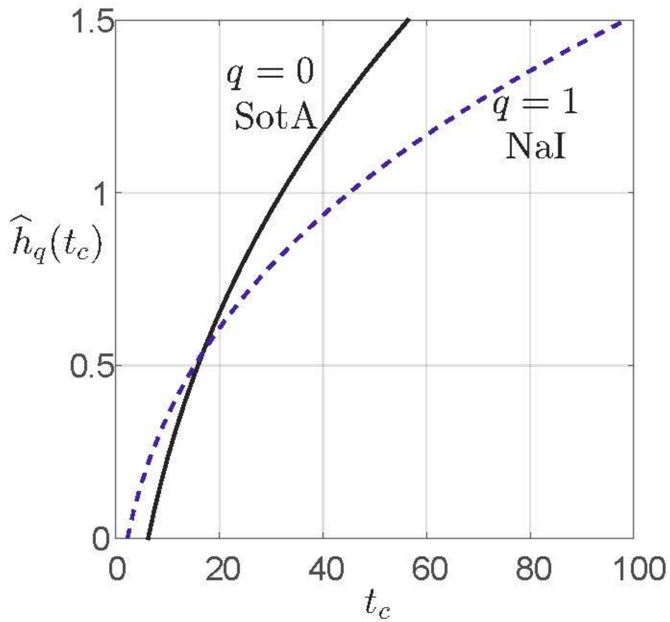
Opportuneness: immunity against windfall.

Minimum necessary uncertainty.

$$\widehat{\beta}_q(t_w) = \min \left\{ h : \ \left(\min_{t,a \in \mathcal{U}(h)} t_q \right) \leq t_w \right\} \qquad (22)$$

Time To Recovery

Robustness vs. Requirement



Trade off: better TTR means worse robustness.

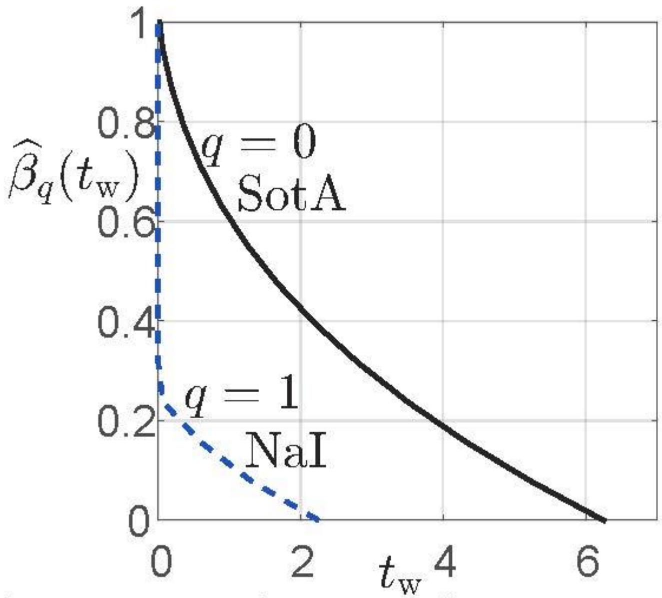
Zeroing: Predicted TTR has zero robustness.

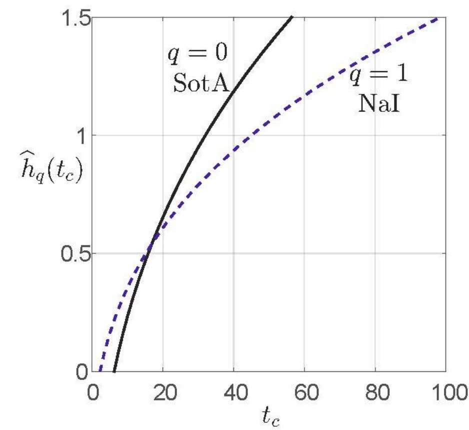
Preference reversal:

- NaI preferred at low TTR. SotA preferred at hi TTR.
- Resolution of innovation dilemma.

Time To Recovery

Opportuneness vs. Aspiration





Trade off: wonderful TTR needs more uncertainty.

Zeroing: Predicted TTR possible without uncertainty.

No preference reversal:

- **No crossing opportuneness curves.**
- **NaI more uncertain and more opportune.**

Time To Recovery: Summary

Task: Recover critical functions in specified time.

Info-gaps:

- Parameter uncertainty: value of a.
- Functional uncertainty: shape of $t(a,q)$.

Innovation dilemma: NaI vs. SotA.

Robustness: maximum tolerable uncertainty.

Opportuneness: minimum required uncertainty.

Trade off, zeroing: robustness and opportuneness.

Optimal monitoring and surveillance:

A paradox of learning

Learning:

- **Discover new knowledge.**
- **Not:** learn French or Newtonian Physics.

Optimal learning:

Min time, max quantity, min cost, max quality…

Monitoring and surveillance as learning:

- New failure mechanism emerging? Where? What?...
- **Not:** does this firm use that amount of power?

Optimal Learning: A Paradox

- **Discover & prevent new failure** with **max effectivity.**
- **Range of design alternatives with fixed resources:**
  – **Extensive** research: **more** knowledge, but less impact.
  – **Limited** research: less knowledge, but **more** impact.
- **Optimal** research amount depends on failure mechanism.
- Failure mechanism is unknown.

Resolution: Satisfice effectivity. **Maximize robustness**.

Procedural (not substantive) **optimization.**

Alternatives: Optimal **adaptive** or **stochastic** learning?

Same paradox of optimal learning.

Same resolution: **robustly satisfice** the design of the learning.

Summing Up

Risk or Uncertainty:

- **Probabilistic** risk, **Knightian** uncertainty (info-gaps).
- Shackle-Popper indeterminism.

Substantive outcome optimization:

Useful under risk, not under uncertainty.

Robust satisficing: Optimize robustness; satisfice goals.

- **Procedural** (not substantive) **optimization.**

Opportune windfalling: use propitious uncertainty.

Time to recovery: Innovation dilemma.

Optimal monitoring and surveillance: A paradox

Questions?