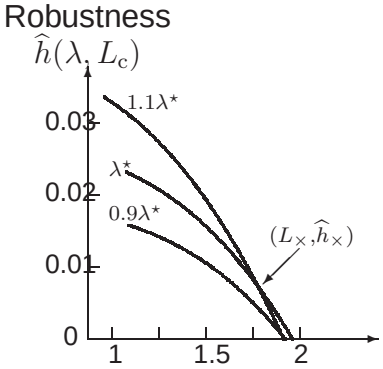

Figure 17: Robustness curves. $\lambda^\star = 3.4065$.

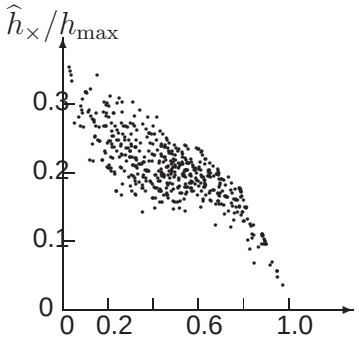

Figure 18: Loci of intersection of robustness curves $\widehat{h}(\lambda^\star, L_c)$ and $\widehat{h}(1.1\lambda^\star, L_c)$.

¶ **Robustness curves** in fig. 17 based on:

- Eqs.(119) and (120).
- Nominal pdf is exponential, $\widetilde{p}(x|\lambda) = \lambda \exp(-\lambda x)$ with $\lambda = 3$.
- Envelope function is constant, $\psi(x) = 1$. Note severe uncertainty on the tail.
- Random sample, X, with $N = 20$.
- MLE of λ, eq.(104): $\lambda^\star = 1/\overline{x}$ where $\overline{x} = (1/N) \sum_{i=1}^{N} x_i$ is the sample mean.
- Robustness curves for 3 λ's: $0.9\lambda^\star$, $\lambda^\star$, and $1.1\lambda^\star$.

¶ **Robustness of the estimated likelihood is zero for any λ:**

- Likelihood function for λ is $L[X, \widetilde{p}(x|\lambda)]$.
- Each curve in fig.17, $\widehat{h}(\lambda, L_c)$ vs. L_c, hits horizontal axis when L_c = likelihood:

$$\widehat{h}(\lambda, L_c) = 0 \quad \text{if} \quad L_c = L[X, \widetilde{p}(x|\lambda)] \tag{121}$$

- $\lambda^\star$ is the MLE of λ. Thus $\widehat{h}(\lambda^\star, L_c)$ hits horizontal axis to the right of $\widehat{h}(\lambda, L_c)$.

¶ **Preferences between estimates of λ:**

- $\widehat{h}(\lambda^\star, L_c) > \widehat{h}(0.9\lambda^\star, L_c) \implies \lambda^\star \succ 0.9\lambda^\star$.
- $\widehat{h}(\lambda^\star, L_c)$ and $\widehat{h}(1.1\lambda^\star, L_c)$ cross at $(L_\times, \widehat{h}_\times)$:
  - $\lambda^\star \succ 1.1\lambda^\star$ for $L_c > L_\times$ and $h < h_\times$.
  - $1.1\lambda^\star \succ \lambda^\star$ else.