

# Info-Gap Theory

## Concepts and Applications

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## Risk or Uncertainty?

Probability is powerful, but ignorance is not probabilistic

## Uncertainty and the optimization imperative

- Limits of prediction and outcome-optimization
- Robust satisficing

## Time to Recovery: Innovation dilemma

## Optimal monitoring and surveillance: A paradox

**Probabilistic risk**  
or  
**Knightian “true uncertainty”**



# Probabilistic Risk

## Consequence

Drought

Industrial accident

Tsunami

Faulty air filters

Deception, scam

## Probability

Stochastic process

Actuarial tables

Historical data

Quality control data

Sociological data

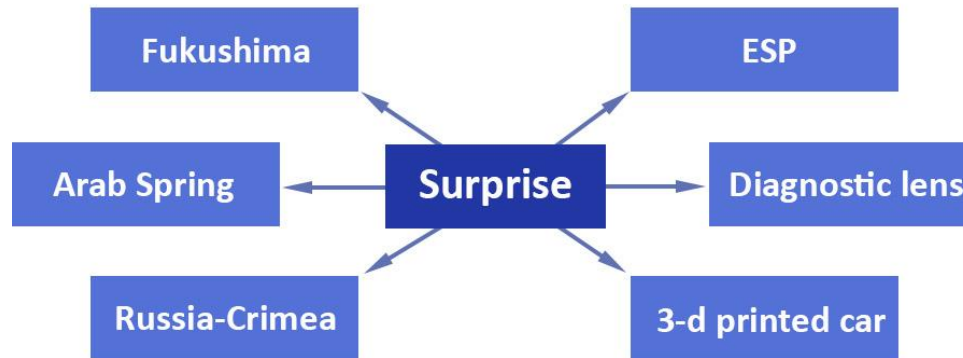


## Risk is:

- Structured: known event space
- Modeled with probability
- Manageable (**but still risky**)

# Frank Knight's "true uncertainty"

"The uncertainties which persist ... are **uninsurable** because there is **no objective measure of the probability**".



# Wheeler's Island

“We live on an island of knowledge surrounded by a sea of ignorance. As our island of knowledge grows, so does the shore of our ignorance.”

*John A. Wheeler*



## D Discovery

- America
- Nuclear fission
- Martians (not yet?)



# Non-probabilistic true uncertainty

## **D** Discovery

## **I** Invention/Innovation

- Printing press: material invention.
- Ecological responsibility: conceptual innovation.
- French revolution: social innovation.





# Non-probabilistic true uncertainty

**D** Discovery

**I** Invention/Innovation

**S** Surprise (Asymmetric uncertainty)

- Ambush
- Competitor's innovation
- Natural catastrophe



# Non-probabilistic true uncertainty

**D** Discovery

**I** Invention/Innovation

**S** Surprise (Asymmetric uncertainty)

What's the next **D** **I** or **S** ???

## **Knightian uncertainty:**

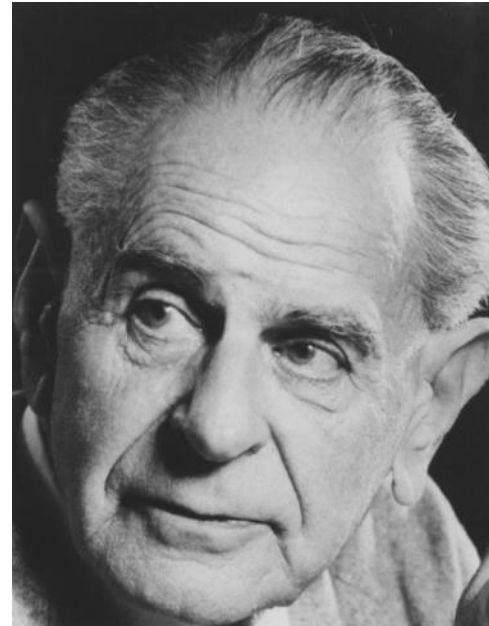
- Unstructured: unknown event space.
- Indeterminate: no laws.
- Barely manageable.

# Shackle-Popper

## Indeterminism



GLS Shackle, 1903-1992



Karl Popper, 1902-1994

# Shackle-Popper Indeterminism

## Intelligence:

What people **know**, influences how they **behave**.



## Discovery:

What will be **discovered tomorrow** can't be **known today**.



Implies

## Indeterminism:

Tomorrow's behavior can't be fully modelled today.

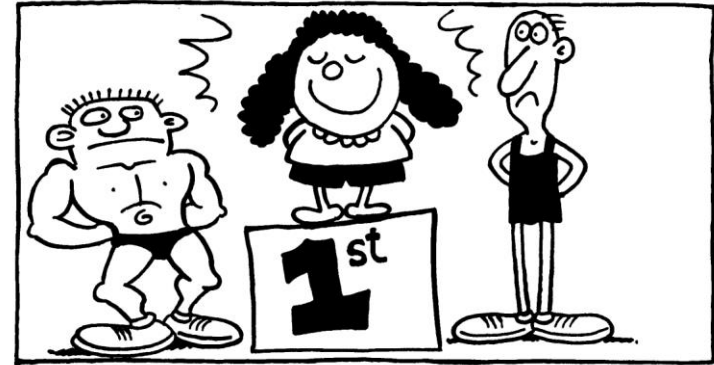
- Info-gaps, indeterminism: **unpredictable**.
- **Ignorance is not probabilistic.**

# Uncertainty and the Optimization Imperative

## Doing your best:

What does that mean?

- Outcome optimization.
- Procedural optimization.



Implications for decision making:  
**Robust satisficing.**



# Doing Your Best

## Substantive outcome optimization:

- Predict outcomes of available options.
- Select predicted best option.

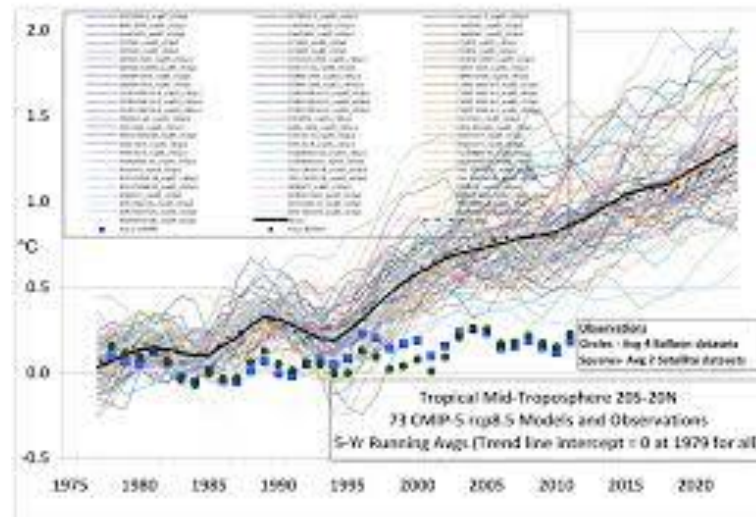


# Doing Your Best

Substantive outcome optimization.

Useful under risk:

- Structured uncertainty.
- Reliable probabilistic predictions.



# Doing Your Best

Substantive outcome optimization:

Useful under **risk**.

Not useful (irresponsible?) under **uncertainty**.

- Unstructured uncertainty.
- Unreliable predictions.



Is this thing plugged in?



# Questions

What do we (**not**) know?



Robustness questions:

- What is an **essential outcome**?
- How to be **robust to surprise**?



Opportuneness questions:

- What is a **windfall outcome**?
- How to **exploit opportunities**?



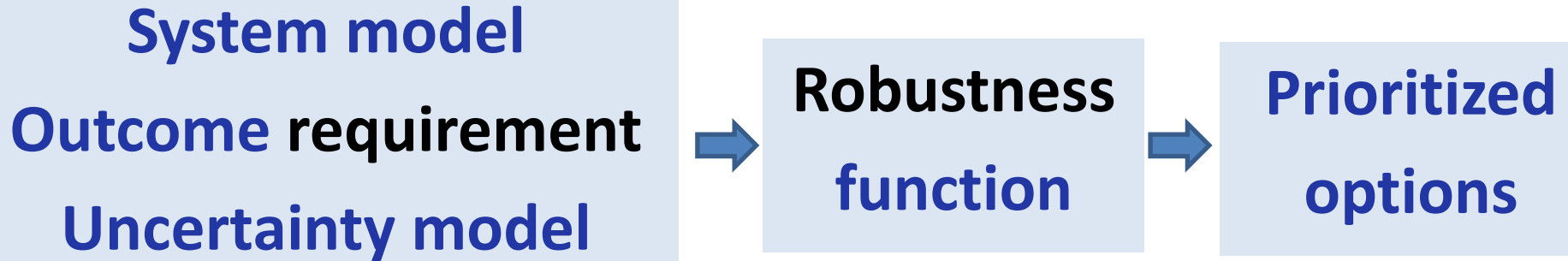
How to prioritize decision options?

What are the trade offs?

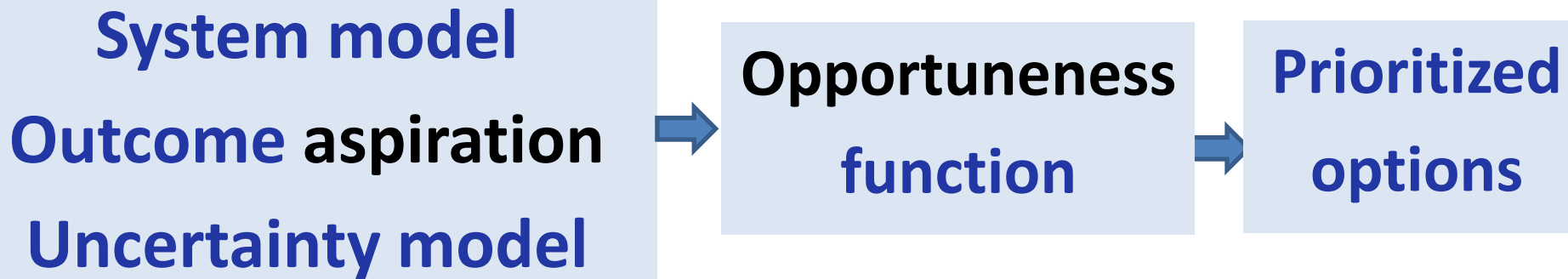


# Answers

## Robustness answer:



## Opportuneness answer:



# Robust Satisficing

## Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?



# Robust Satisficing

## Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

## 1. Satisficing: Achieving critical outcomes.

- Essential goals.
- Worst acceptable outcomes.
- Modest or ambitious.



# Robust Satisficing

## Two questions for decision makers:

1. What are our goals?
2. How much error/surprise can we tolerate?

## 1. Satisficing: Achieving critical outcomes.

## 2. Robustness:

- Immunity to ignorance.
- Greatest tolerable error or surprise.

# Robust Satisficing

**Two questions for decision makers:**

1. What are our goals?
2. How much error/surprise can we tolerate?

**1. Satisficing: Achieving critical outcomes.**

**2. Robustness: Greatest tolerable error.**

**Optimize robustness; satisfice goals:**

**Procedural (not substantive) optimization.**

# Time To Recovery

Time to recovery (TTR) after disruption:

- Building after earthquake.
- Energy distribution network after failure.
- Micro-sensor after shock load.
- Etc.

Task: Recover critical functions in specified time.

Challenge: **Uncertainties** (info-gaps).

# Time To Recovery

Formulation: Innovation dilemma.

- Choose between 2 design concepts:
  - State of the art (SotA,  $q=0$ ).
  - New and innovative (Nai,  $q=1$ ).
- System model: TTR,  $t(a,q)$ , to load  $a$  for system  $q$ .
- Outcome requirement and aspiration:  
 $t(a,q) < tc$ ,       $t(a,q) < tw \ll tc$

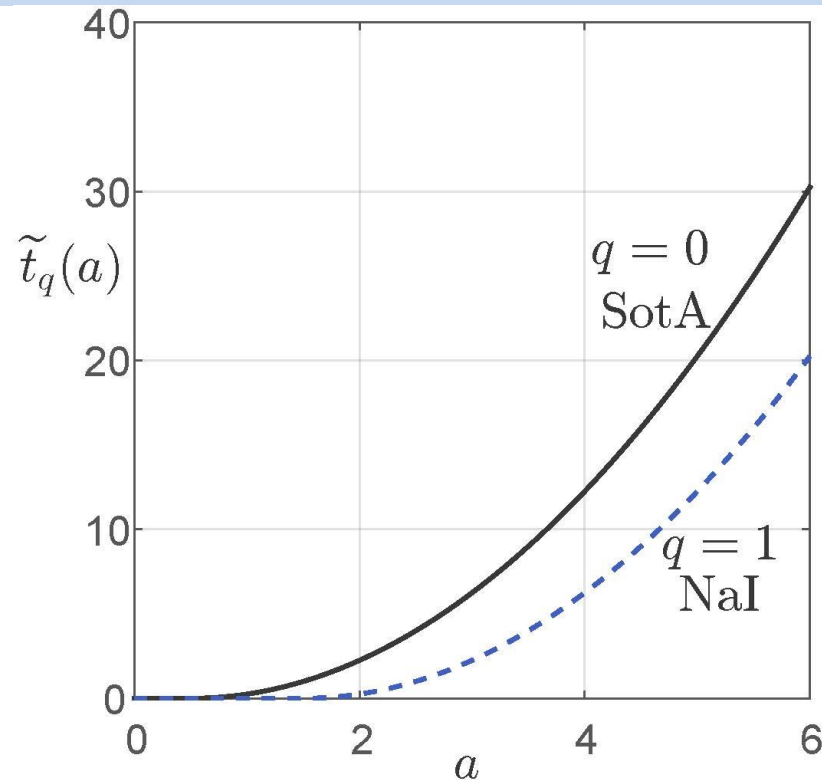
## Info-gaps:

- Parameter uncertainty: value of  $a$ .
- Functional uncertainty: shape of  $t(a,q)$ .



# Time To Recovery

Estimated TTR functions for 2 designs.



Putative preference: NaI predicted better than SotA.

What about **uncertainty** in load  $a$  & TTR func  $t(a,q)$ ?

# Time To Recovery

Info-gap:

Disparity between what we do know (on  $a$  &  $t(a,q)$ ) and what we need to know in order to make responsible decision (SotA or NaI).

About the load,  $a$ :

**Known** estimated value. **Unknown** fractional error.

About the TTR function,  $t(a,q)$ :

- **Known** estimated form. **Unknown** fractional error.
- NaI **more uncertain** than SotA.

# Time To Recovery

Info-gap model of uncertain  $a$  and  $t(q,a)$ :

$$\mathcal{U}(h) = \left\{ a, t_q(a) : t_q(a) \geq 0, \quad |t_q(a) - \tilde{t}_q(a)| \leq hw_q \tilde{t}_q(a), \quad q = 0, 1. \right. \\ \left. a > 0, \quad \left| \frac{a - \tilde{a}}{s} \right| \leq h \right\}, \quad h \geq 0 \quad (18)$$

- **Non-prob: unbounded family of nested sets.**
- **Horizon of uncertainty,  $h$ , unknown.**
- **No known worst case.**
- **Axioms: Contraction and Nesting.**

# Time To Recovery

Immunity functions.

**Robustness: immunity against failure.**

**Maximum tolerable uncertainty.**

$$\widehat{h}_q(t_c) = \max \left\{ h : \left( \max_{t, a \in \mathcal{U}(h)} t_q \right) \leq t_c \right\} \quad (21)$$

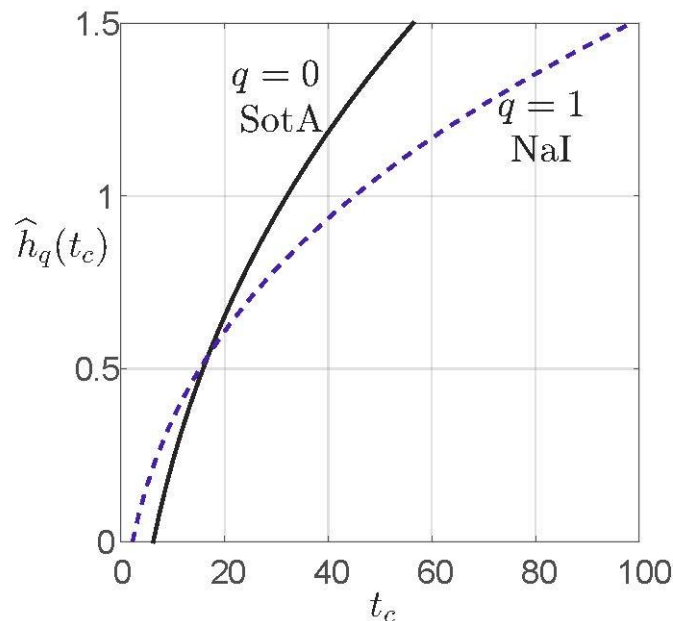
**Opportuneness: immunity against windfall.**

**Minimum necessary uncertainty.**

$$\widehat{\beta}_q(t_w) = \min \left\{ h : \left( \min_{t, a \in \mathcal{U}(h)} t_q \right) \leq t_w \right\} \quad (22)$$

# Time To Recovery

## Robustness vs. Requirement



Trade off: better TTR means worse robustness.

Zeroing: Predicted TTR has zero robustness.

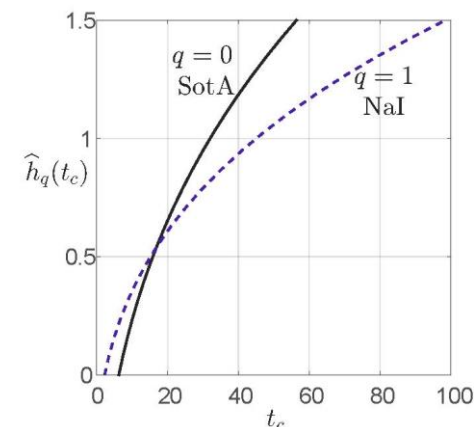
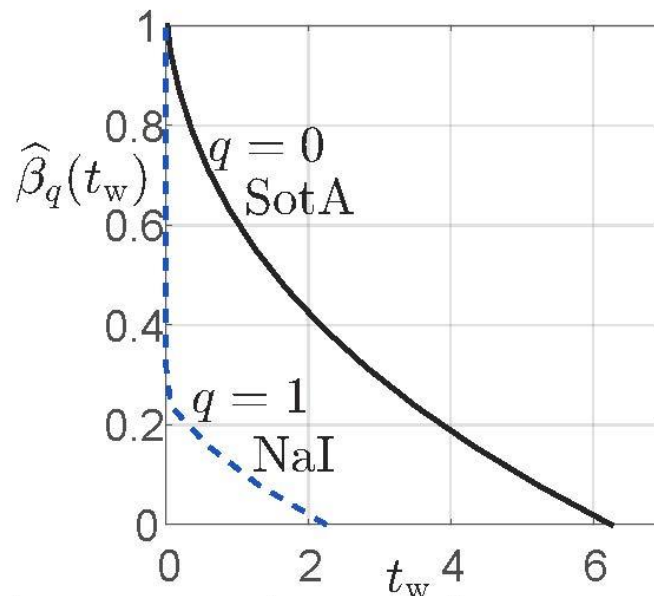
Preference reversal:

- NaI preferred at low TTR. SotA preferred at hi TTR.
- Resolution of innovation dilemma.

# Time To Recovery



## Opportuneness vs. Aspiration



**Trade off: wonderful TTR needs more uncertainty.**

**Zeroing: Predicted TTR possible without uncertainty.**

**No preference reversal:**

- No crossing opportuneness curves.**
- NaI more uncertain and more opportune.**

# Time To Recovery: Summary

Task: Recover critical functions in specified time.

## Info-gaps:

- Parameter uncertainty: value of  $a$ .
- Functional uncertainty: shape of  $t(a, q)$ .

Innovation dilemma: NaI vs. SotA.

Robustness: maximum tolerable uncertainty.

Opportuneness: minimum required uncertainty.

Trade off, zeroing: robustness and opportuneness.

# Optimal monitoring and surveillance:

## A paradox of learning

### Learning:

- Discover new knowledge.
- **Not:** learn French or Newtonian Physics.



### Optimal learning:

Min time, max quantity, min cost, max quality...

### Monitoring and surveillance as learning:

- New failure mechanism emerging? Where? What?...
- **Not:** does this firm use that amount of power?



# Optimal Learning: A Paradox

- Discover & prevent new failure with **max effectivity**.
- Range of design alternatives with fixed resources:
  - **Extensive** research: **more** knowledge, but **less** impact.
  - **Limited** research: **less** knowledge, but **more** impact.
- **Optimal** research amount depends on failure mechanism.
- **Failure mechanism is unknown.**

**Resolution: Satisfice** effectivity. **Maximize robustness.**

**Procedural** (not substantive) **optimization.**

**Alternatives:** Optimal **adaptive** or **stochastic** learning?

Same **paradox** of optimal learning.

Same resolution: **robustly satisfice** the design of the learning.

# Summing Up

## Risk or Uncertainty:

- **Probabilistic** risk, **Knightian** uncertainty (info-gaps).
- Shackle-Popper indeterminism.

## Substantive outcome optimization:

Useful under **risk**, not under **uncertainty**.

**Robust satisficing:** Optimize robustness; satisfice goals.

- **Procedural** (not substantive) **optimization**.

**Opportune windfalling:** use propitious uncertainty.

**Time to recovery:** Innovation dilemma.

**Optimal monitoring and surveillance: A paradox**

# Questions?

