Robust Satisficing Voting
Why are uncertain voters biased towards sincerity?

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Abstract

The modern theory of voting usually regards voters as expected utility maximizers. This implies that voters define subjective probabilities and utilities for different outcomes of the elections. In real life, these probabilities and utilities are often highly uncertain, so a robust choice, immune to erroneous assumptions, may be preferred. We show that a voter aiming to satisfice his expected utility, rather than maximize it, may present a bias for sincere voting, as opposed to strategic voting. This may explain previous results which show that strategic voting is not as prevalent as would be expected if all voters were expected utility maximizers.

1 Introduction

The modern theory of voting usually regards voters as rational in the sense of *expected utility maximization* (McKelvey and Ordeshook 1972). This model is subject to criticism arising from paradoxes such as those of Ellsberg and Allais. Furthermore, this model does not explain why cases of non-sincere voting (in the sense defined by Farquharson 1969 and to be discussed subsequently) are rare, even when voters are expected to benefit from such voting patterns (Herzberg and Wilson 1988; Eckel and Holt 1989; Blais and Nadeau 1996). We will define an alternative model, in which voters aim to *satisfice* the expected utility rather than *maximize* it. We will show that this model suggests a bias against non-sincere voting.

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A major focus of voting theory research, from the very cradle of this science, was on the prevention of paradoxical election results (for instance, when the Condorcet loser is elected by the majority of voters), and to prevent voters from insincere misrepresentation of their preferences (Black 1963; Saari 2001a). A seminal result, Arrow's theorem of impossibility (Arrow 1950), proves that paradoxical results may be an inherent property of any process of social choice, and elections are no different (Saari 2001b).

Arrow (1950), as well as other researchers (see Black, 1963), assumed the decision process receives the *a priori* preferences of the voters as its input. That is, the decision process is based on preferences of the voters between the different alternatives, without considering the likelihood of different outcomes of the decision process. Farquharson (1969) defined this kind of voting as *sincere*, while acknowledging that both the decision process (or agenda) and the voting may be manipulated by sophisticated participants. He coined the term *sophisticated voter*, which referred to voters who vote not according to their *a priori* preferences among the candidates, but in the way that will yield the most favorable possible result. The term *strategic voting* was later used to define voting which does not reflect *a priori* preferences. That is, strategic voting occurs when a voter benefits from voting "insincerely" based on some global criterion. Although neither Arrow (1950) nor Farquharson (1969) considered probabilistic settings (Arrow considered deterministic settings, while Farquharson considered dominance of alternatives), expected utility is often used as such a criterion.

The possibility of insincere (strategic) voting was familiar long before the 20th century, and was widely regarded as unwanted. Charles Dodgson, for example, said that "... it is better for elections to be decided according to the wish of the majority than of those who happen to have most skill in the game..." (Black 1963: 233). However, the Gibbard-Satterthwaite theorem (Gibbard 1973; Satterthwaite 1975) proves that for most social decision procedures there is a constellation in which some voter may profit from voting insincerely. In fact, the only decision system immune to strategic voting is a dictatorship. See Taylor (2005) for a comprehensive review of the manipulation of voting systems.

Although both issues, paradoxical results and the possibility of strategic voting, have proved to be impossible to resolve, the debate over the "best" voting procedure continues even today. In this debate, both normative, empirical, and theoretical arguments are used to advocate one of the old classics, such as the Borda count (Saari 2001b), or a newer procedure, such as approval voting (Brams 2008).

Strategic voting is more than a theoretical concept, relevant only in extreme and rare situations. There are ample examples of strategic voting in recent history. Chen and Yang (2002) give examples of strategic voting in open primaries in Taiwan and in the United States. Abramson *et al*. (2004) suggest that in the Israeli election for Prime Minister in 1999, one of the candidates withdrew because of the expected strategic voting of his supporters. Johnson *et al*. (2005) concluded that the Chief Justice in the United States Supreme Court votes strategically, since his seniority allows him to assign the judge who will write the majority decision, assuming that he is part of the majority.

Strategic voting depends on knowledge of other voters' strategies (see also Chopra *et al*. 2004). This knowledge is not reliably available in most real life decision situations. The standard method for modeling this uncertainty is probabilistic. Voting games are often compared to game-theory games with incomplete information (Harsanyi 1967), or global games (Carlsson and van Damme 1993; Morris and Shin 2001). In the case of games of incomplete information, there is (probabilistic) uncertainty regarding the utilities of the other players and their available strategies, where the probability distributions are known to the players. Global games broaden this concept, by stating that the entire game (including the player's utilities) are chosen randomly from a known set of possible games.

But what is the extent of strategic voting? Research has shown that strategic (non-sincere) voting is not that prevalent. Herzberg and Wilson (1988) conducted an experiment in committee voting procedure where only 20%-40% of the voters displayed a strategic, non-sincere, behavior, while all voters would have benefited from such behaviour. Following their own experiment in committee voting, Eckel and Holt (1989) concluded that strategic voting will become prevalent only with experience, that is, when the procedure is repeated and the preferences of the other voters becomes clear.

Blais and Nadeau (1996) analyzed the results of the Canadian elections in 1988, which were ternary (tripartite) by nature. They discovered that only 6% of the voters voted strategically, while 20% of the population had the chance to. This contradicts the conjecture of Duverger's law, which states that "the simple majority, single ballot system favors the two-party system" (Riker 1982). In other words, in order not to waste their votes, supporters of third (or smaller) parties would prefer to vote for one of the larger parties. Blais (2004) suggested that the low incidence of strategic voting is due to high intensity of preference (the most preferred party is significantly preferred to the second-best party) and a bias in the third party's estimated chances of winning. We will suggest a different explanation to these low percentages, which will be based on info-gap decision theory (Ben-Haim 2006).

We will show that if the voters aim to *satisfice* their expected utility, rather than *optimize* it, then there is a bias towards sincere voting. That is, voters who attempt to satisfice the outcome will be more similar to sincere voters than voters who attempt to optimize. The motivation for satisficing (rather than optimizing) derives from the great uncertainty associated with estimates of the voters' subjective probabilities and utilities. We will demonstrate the irrevocable trade-off between robustness to this uncertainty on the one hand, and expected utility on the other. A decision which attempts to maximize the expected utility is a decision with zero robustness to uncertainty in the subjective probabilities. A decision which satisfices the expected utility has positive robustness. Satisficing strategies are biased towards sincere voting as opposed to strategic voting.

The paper is organized as follows. Section 2 briefly describes how info-gap theory is used to robustly satisfice a requirement. Section 3 proves that robust-satisficing voting is biased towards sincere voting for ternary elections under plurality voting. Section 4 generalizes this result. A concluding discussion appears in Section 5. Mathematical details appear in appendices.

2 Info-Gap Theory: Précis

Decision making is often viewed as choosing a decision q from a set $\mathcal{Q}$ of possible decisions, that will maximize some reward function $R(q)$. However, more often than not, the reward function must take into account various things which are unknown to the decision maker. These unknowns may be the value of some parameter, or even the functional relation between the decision q and the reward $R(q)$ (the model itself), or a set of parameters or functions. We will refer to these parameters and models as the state of the world. We will denote the reward function as $R(q, u)$, where u is the state of the world and $\mathcal{U}$ the set of possible states of the world. We may have a best estimate $\widetilde{u}$ of the state of the world, but our uncertainty around the true state of the world is non-probabilistic. In many cases, it is also unbounded.

Info-gap models are used to quantify non-probabilistic "true" (Knightian) uncertainty (Ben-Haim 2006). An info-gap model is an unbounded family of nested sets, $\mathcal{U}(\alpha, \widetilde{u})$. At any level of uncertainty α, the set $\mathcal{U}(\alpha, \widetilde{u})$ contains possible realizations of u. As the horizon of uncertainty α gets larger, the sets become more inclusive. The info-gap model expresses the decision maker's beliefs about uncertain variation of u around $\widetilde{u}$.

Info-gap models obey two axioms:

$$\text{Contraction:} \qquad \mathcal{U}(0, \widetilde{u}) = \{\widetilde{u}\} \tag{1}$$

$$\text{Nesting:} \qquad \alpha < \alpha' \text{ implies } \mathcal{U}(\alpha, \widetilde{u}) \subseteq \mathcal{U}(\alpha', \widetilde{u}) \tag{2}$$

The contraction axiom asserts that $\widetilde{u}$ is the only possibility when there is no uncertainty ($\alpha = 0$). The nesting axiom asserts that the range of possible realizations increases as the level of uncertainty increases. The value of α is unknown, meaning that there is no known worst case.

Suppose the decision maker does not only wish the reward to be as high as possible, but has some notion of a critical value r_c. This means that a reward higher than r_c would be appreciated, but a reward smaller than r_c cannot be tolerated.

The *robustness function* $\widehat{\alpha}(q, r_c)$ is the greatest horizon of uncertainty α at which reward no smaller than r_c is guaranteed:

$$\widehat{\alpha}(q, r_c) = \max \left\{ \alpha : \left(\min_{u \in \mathcal{U}(\alpha, \widetilde{u})} R(q, u) \right) \geq r_c \right\} \tag{3}$$

The robustness can be evaluated even though there is no known worst case. Furthermore, the robustness function generates preferences on the decisions, q: a decision which is more robust for achieving aspiration r_c is preferred over a decision which is less robust. *Robust-satisficing* decision making maximizes the robustness and satisfices the reward at the value r_c, without specifying a limit on the level of uncertainty:

$$\widehat{q} = \arg\max_{q \in \mathcal{Q}} \widehat{\alpha}(q, r_c) \tag{4}$$

where $\mathcal{Q}$ is the set of available decisions.

It can readily be shown that there is an inherent trade-off between robustness and performance (Ben-Haim 2006). Since robustness is the *immunity to failure*, the robustness decreases as the performance requirement r_c becomes more demanding. Another immediate result is that the robustness of the *optimal* result—the maximal reward under our best estimate $\widetilde{u}$—has zero robustness, meaning that a slight deviation from our estimate $\widetilde{u}$ may prevent us from meeting the requirement r_c.

3 Robust Satisficing for Three-Candidate Plurality Voting

3.1 Strategic Voting vs. Robust-Satisficing Voting

Rational voting, in the sense defined by McKelvey and Ordeshook (1972), under plurality elections (without runoff) is based on expected marginal utility. This model takes into account the marginal utility of influencing the outcomes of the elections, and the probability of influencing the outcomes in different contexts. The rational voter maximizes the expected marginal utility, and may vote for an alternative with individual utility (as opposed to expected utility) which is lower than the utility of some other alternative, *i.e.*, strategically. The "sincere" voter votes for the option whose utility is maximal, without considering the expected outcome of the election as a whole. In this section we will discuss three propositions which suggest why strategic—as opposed to sincere—voting is less common than might be expected.

We will denote the utility for some voter from the winning of candidate i by u_i. The voter has some subjective probability that candidates i and j would be tied, and that her vote would decide the winner of the election. We will denote this subjective probability by p_{ij}. Then the expected marginal utility for the voter from voting for candidate i is:

$$U_i = \sum_{j \neq i} p_{ij} \Delta_{ij} \tag{5}$$

where $\Delta_{ij} = u_i - u_j$ (Myerson and Weber 1993). Note that if we do not demand $\sum_i \sum_{j \neq i} p_{ij} = 1$, then the voter does not assume that he will necessarily cast the decisive vote. Also, notice that the above model neglects the probability of three-way (or more) ties. This is since we assume, after McKelvey and Ordeshook (1972), that the probabilities for ties between three or more candidates are negligible.

If the voter is an expected utility maximizer, then he will vote for the candidate for which the expected marginal utility, U_i, is maximal. Assume that the voter's sincere preferences are $u_x > u_y > u_z$ and so $\Delta_{xy}, \Delta_{xz}, \Delta_{yz} > 0$. An expected utility maximizing voter would vote strategically—that is, unlike the sincere vote (which is for x)—only if the expected marginal utility from voting for x is lower than the expected marginal utility from voting for y, namely, only if $U_x < U_y$. In other words, such a voter will vote strategically for y if and only if:

$$p_{xy}\Delta_{xy} + p_{xz}\Delta_{xz} < -p_{xy}\Delta_{xy} + p_{yz}\Delta_{yz} \tag{6}$$

Notice that:

$$U_z = -p_{xz}\Delta_{xz} - p_{yz}\Delta_{yz} \leq 0 \leq p_{xy}\Delta_{xy} + p_{xz}\Delta_{xz} = U_x \tag{7}$$

so voting for z is never an option.

The probabilities p_{ij} are subjective, but they are also based on noisy signals the voter receives from her environment: polls, commentary, friends' opinions, and so on. These probabilities are therefore highly uncertain. We will describe this uncertainty through an info-gap model:

$$\mathcal{U}(\alpha, \widetilde{p}) = \left\{ p \,:\, \forall i \neq j \,,\; \left| \frac{p_{ij} - \widetilde{p}_{ij}}{\widetilde{p}_{ij}} \right| \leq \alpha \,,\; p_{ij} \geq 0 \,,\; \sum_{j \neq i} p_{ij} \leq 1 \right\} \quad , \quad \alpha \geq 0 \tag{8}$$

Here, $\widetilde{p}_{ij}$ represents the voter's subjective probabilities, which are her best estimate of the "actual" probabilities. Since p_{ij} and p_{kl} represent disjoint events, the sum of their probabilities is at most 1 but need not equal 1. Notice that $\sum_{i \neq j} \widetilde{p}_{ij}$ is the subjective probability of the voter casting the decisive vote, and it is actually reasonable to assume that $\sum_{i \neq j} \widetilde{p}_{ij} \ll 1$. We shall discuss uncertainties in the utilities from the candidates later on.

Assume, without loss of generality, that the sincere preference of the voter is $u_x > u_y > u_z$. Based on the info-gap model of eq. (8), we may now define robustness functions for the two voting alternatives, x and y. The robustness function $\widehat{\alpha}(x, u_c)$ is the greatest horizon of uncertainty in the probabilities, at which the expected marginal utility of voting for x is no less than the critical utility u_c. The formal definition for this robustness function is:

$$\widehat{\alpha}(x, u_c) = \max \left\{ \alpha : \left(\min_{p \in \mathcal{U}(\alpha, \widetilde{p})} U_x \right) \geq u_c \right\} \quad (9)$$

$\widehat{\alpha}(y, u_c)$ is similarly defined with respect to voting for y: the greatest horizon of uncertainty in the probabilities, at which the expected marginal utility of voting for y is no less than the critical utility u_c. The formal definition for this robustness function is:

$$\widehat{\alpha}(y, u_c) = \max \left\{ \alpha : \left(\min_{p \in \mathcal{U}(\alpha, \widetilde{p})} U_y \right) \geq u_c \right\} \quad (10)$$

It is possible to give an explicit functional form for these robustness functions assuming that $\widetilde{p}_{xy} < \frac{1}{2}$, which is entirely reasonable since $\sum_{i \neq j} \widetilde{p}_{ij} \ll 1$ (see Appendix A):

$$\widehat{\alpha}(x, u_c) = \begin{cases} \infty & \text{, if } u_c \leq 0 \\ 1 - \dfrac{u_c}{\widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{xz}\Delta_{xz}} & \text{, if } 0 < u_c \leq \widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{xz}\Delta_{xz} \\ 0 & \text{, if } u_c > \widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{xz}\Delta_{xz} \end{cases} \quad (11)$$

$$\widehat{\alpha}(y, u_c) = \begin{cases} \infty & \text{, if } u_c \leq -\Delta_{xy} \\ \dfrac{-\widetilde{p}_{xy}\Delta_{xy} - u_c}{\widetilde{p}_{xy}\Delta_{xy}} & \text{, if } -\Delta_{xy} < u_c \leq -2\widetilde{p}_{xy}\Delta_{xy} \\ \dfrac{-\widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{yz}\Delta_{yz} - u_c}{\widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{yz}\Delta_{yz}} & \text{, if } -2\widetilde{p}_{xy}\Delta_{xy} < u_c \leq -\widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{yz}\Delta_{yz} \\ 0 & \text{, if } u_c > -\widetilde{p}_{xy}\Delta_{xy} + \widetilde{p}_{xz}\Delta_{xz} \end{cases} \quad (12)$$

A robust-satisficing voter would vote for the candidate with the higher robustness. In other words, a robust-satisficing voter would vote strategically only if $\widehat{\alpha}(y, u_c) > \widehat{\alpha}(x, u_c)$.

3.2 Robust-Satisficing Voting and the Bias Towards Sincerity

We will now prove that robust-satisficing voting is biased towards sincere voting. That is, robust-satisficing voters will tend to vote strategically less than expected utility maximizing voters. Whenever an expected utility maximizing voter would have voted sincerely, a robust-satisficing voter will also vote sincerely. However, when an expected utility maximizing voter would have voted strategically for the second-best alternative (due to its higher estimated expected utility), a robust-satisficing voter may still vote sincerely.

Recall that an expected utility maximizing voter will vote for candidate x if $U_x > U_y$ and $U_x > U_z$.

Proposition 1 *Robust-satisficing voting in a three-candidate plurality election is biased towards sincere voting.*
Given *that the sincere preference of a voter is $u_x > u_y > u_z$ and the uncertainty is defined by the info-gap model of eq.* (8), **if** *the expected utility maximizing vote is for x,* **then** *the robust-satisficing vote is also for x,* **and if** *the expected utility maximizing vote is for y,* **then** *there exists a range of critical values u_c for which the robust-satisficing vote is for x.*

Proposition 1 asserts that uncertainty in the probabilities, when it induces the robust-satisficing strategy, results in a bias for sincere voting as against strategic voting.

Our second proposition generalizes the above result for a situation where both subjective probabilities and utilities are uncertain. Uncertainty in the utilities may arise when the candidates are unclear on some controversial issue, or if voters are uncertain of candidates' ability (or will) to implement their pronounced agenda. Even if the candidates are both clear and trustworthy, it is difficult to assign an accurate numerical utility to each of the candidates.

First, we have to define an appropriate info-gap model. The following info-gap model corresponds to the case where the both subjective probabilities and utilities of the candidates are uncertain:

$$\mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta}) = \left\{ p, \Delta \ : \ \forall i \neq j \ , \quad \begin{array}{c} \left|\frac{p_{ij} - \widetilde{p}_{ij}}{\widetilde{p}_{ij}}\right| \leq \alpha \ , \ p_{ij} \geq 0 \ , \ \sum_{j \neq i} p_{ij} \leq 1 \\ \left|\frac{\Delta_{ij} - \widetilde{\Delta}_{ij}}{\widetilde{\Delta}_{ij}}\right| \leq h\alpha \ , \ \Delta_{xy} + \Delta_{yz} = \Delta_{xz} \end{array} \right\} \quad , \quad \alpha \geq 0 \tag{13}$$

where $h > 0$ is a calibration factor, allowing for "different degrees of uncertainty" between probabilities and utilities. Notice that we do not demand $\Delta_{ij} \geq 0$, since it may turn out that the "true" order of preference as implied by Δ (the actual utilities) differs from the estimated order of preference as implied by $\widetilde{\Delta}$.

We can now define robustness functions as in eqs. (9)-(10) based on this info-gap model rather than on eq. (8).

Proposition 2 *Robust-satisficing voting in a three-candidate plurality election is biased towards sincere voting when the uncertainty revolves around both the subjective probabilities and the utilities of the candidates.*

Given *that the estimated sincere preference of a voter is* $\widetilde{u}_x > \widetilde{u}_y > \widetilde{u}_z$*, the voter's uncertainty is defined by the info-gap model of eq.* (13)*, and the voter will only consider positive critical utility values,* **if** *the expected utility maximizing vote is for* x*,* **then** *the robust-satisficing vote is also for* x*,* **and if** *the expected utility maximizing vote is for* y*,* **then** *there exists a range of critical values* u_c *for which the robust-satisficing vote is for* x*.*

We have demonstrated that, compared to the expected utility maximizing voter, a robust-satisficing voter is biased towards sincere voting. This bias is consistent with the results of Blais and Nadeau (1996), and might also be able to explain the results of Hertzberg and Wilson (1988) and Eckel and Holt (1989).

However, the above results refer only to ternary elections under plurality voting. In the following section we will show that these results apply to many other constellations.

4 Robust-Satisficing Voting Bias: Generalizing the Results

4.1 Plurality Voting with n Candidates

Assume there are n candidates, $i_1, \ldots, i_n$, where $u_{i_1} > \cdots > u_{i_n}$. Using the notations of the previous section, the expected utility from voting for candidate i is:

$$U_i = \sum_{j \neq i} p_{ij} \Delta_{ij} \tag{14}$$

For the sake of simplicity, we will focus on uncertainty surrounding the subjective uncertainties, $\widetilde{p}_{ij}$. The info-gap model is therefore:

$$\mathcal{U}(\alpha, \widetilde{p}) = \left\{ p \ : \ \forall i \neq j \ , \ \left| \frac{p_{ij} - \widetilde{p}_{ij}}{\widetilde{p}_{ij}} \right| \leq \alpha \ , \ p_{ij} \geq 0 \ , \ \sum_{j \neq i} p_{ij} \leq 1 \right\} \quad , \quad \alpha \geq 0 \tag{15}$$

We will also assume that $\sum_{j \neq i} \widetilde{p}_{ij} \ll 1$.

It is now possible to prove that robust-satisficing voting is biased towards sincere voting.

Proposition 3 *Robust-satisficing voting in an* n*-candidate plurality election is biased towards sincere voting.*
Given *that the sincere preference of a voter is* $u_{i_1} > \cdots > u_{i_n}$ *and the uncertainty is defined by the info-gap model of eq.* (15)*,* **if** *the expected utility maximizing vote is for* i_1*,* **then** *the robust-satisficing vote is also for* i_1*,* **and if** *the expected utility*

maximizing vote is for $j \neq i_1$, **then** *there exists a range of critical values u_c for which the robust-satisficing vote is for i_1.*

Proposition 3 proves a bias towards the most preferable candidate. However, it does not guarantee a "total" bias. That is, a robust-satisficing voter is not necessarily biased towards preferring candidate i_2 over candidate i_3.

Total bias can be achieved by a stricter set of assumptions. Assume that for every candidate there is a subjective probability $\widetilde{q}_i$, where the subjective probability of casting the decisive vote between i and j is $\widetilde{p}_{ij} = \widetilde{q}_i \widetilde{q}_j$. We will denote this assumption as *independence of subjective probabilities*

Notice that the above restriction applies only to $\widetilde{p}$, not to the info-gap model. Therefore, we may still use the info-gap model of eq. (15).

Proposition 4 *Robust-satisficing voting in an n-candidate plurality election is totally biased towards sincere voting.*
Given *that the sincere preference of a voter is $u_{i_1} > \cdots > u_{i_n}$, the uncertainty is defined by the info-gap model of eq.* (15) *and there is independence of subjective probabilities,* **if** *$u_i > u_j$ and $\widetilde{U}_i \geq \widetilde{U}_j$,* **then** *$\widehat{\alpha}(i, u_c) \geq \widehat{\alpha}(j, u_c)$ for $u_c \geq 0$,* **and if** *$u_i < u_j$ and $\widetilde{U}_i \geq \widetilde{U}_j > 0$,* **then** *there exists a non-negative range of critical values u_c for which $\widehat{\alpha}(i, u_c) \leq \widehat{\alpha}(j, u_c)$.*

4.2 Different Voting Systems

The results of Propositions 3-4 have direct implications on voting systems other than plurality voting. We will discuss here two such systems, the Borda count (Saari 2001b) and approval voting (Brams 2008). We will not develop an explicit form for either the expected utilities or the robustness functions under these voting systems, as the derivation is both straightforward and lengthy.

Under the Borda count, each voter assigns n points to its most preferred candidate, $n-1$ points to his second favorite candidate, and so on. In order to prove bias towards sincerity, we must first be able to tell which vote is more sincere.

Let $b_1, \ldots, b_n$ denote the score given by the voter to candidates $i_1, \ldots, i_n$. Let $c_1, \ldots, c_n$ be another possible score for this voter, where:

$$c_j = \begin{cases} b_j & \text{, if } j \neq k, l \\ b_k & \text{, if } j = l \\ b_l & \text{, if } j = k \end{cases} \tag{16}$$

where $k \neq l$. We will say that b is more sincere than c if $u_{i_k} > u_{i_l}$ and $b_k > c_k$. That is, if b gives higher score to the more favorable candidate. The following corollary predicts bias towards sincerity for robust-satisficing voting under the Borda count.

Corollary 5 *Robust-satisficing voting under Borda count is totally biased towards sincere voting.*
Given *score b, which is more sincere than score c, the uncertainty is defined by the info-gap model of eq.* (15) *and that there is independence of subjective probabilities,* **if** *score b has higher expected utility than score c,* **then** *the robust-satisficing vote is also b,* **and if** *score c has higher expected utility than score b,* **then** *there exists a range of critical values u_c for which the robust-satisficing vote is b.*

Notice that we proved total bias under the assumption of independence of subjective probabilities. Without this assumption we can only prove bias towards giving the voter's most preferred candidate the maximal n points.

Under approval voting each voter assigns to each candidate either a single point (1, or "approve") or zero points (0, or "do not approve"). Similarly to the Borda count, we will denote a vote by $b_1, \ldots, b_n$. There are several possible sincere votes. A vote is considered strategic if $b_k = 0$ (candidate i_k not approved), $b_l = 1$ (candidate i_l approved) and $u_{i_k} > u_{i_l}$. The following corollary predicts bias towards sincerity for robust-satisficing voting under approval voting.

Corollary 6 *Robust-satisficing voting under approval voting is biased towards sincere voting.*
Given *that the uncertainty is defined by the info-gap model of eq.* (15), **if** *c is a strategic vote,* **then** *there exists a sincere vote b such that:*

- **if** *vote b has higher expected utility than vote c,* **then** *the robust-satisficing vote is also b,*
- **and if** *vote c has higher expected utility than vote b,* **then** *there exists a range of critical values u_c for which the robust-satisficing vote is b.*

We have shown that robust-satisficing is biased towards sincerity for voting systems other than plurality voting. The lines of proofs may applied to yet other voting systems.

It is worth noting that while we have shown that both Borda count and approval voting are biased toward sincerity voting under robust-satisficing voting, they are not necessarily *equally biased*. It is possible that one of the systems would be "more biased" than the other. For instance, consider the point of preference reversal, the highest critical value for which a robust-satisficing voter would vote sincerely when an expected utility maximizer would have voted strategically (see Figure 1b). If the point of preference reversal tends to be closer to the nominal expected utility ($\widetilde{U}$) for one of the voting systems we may conclude that it is more biased towards sincerity.

Borda count and approval voting are quite different in their amenability to manipulation, although there is no consensus over which is more likely to induce sincere voting (see, for instance, Saari 2001b and Brams 2008). We have shown that both these systems also are biased towards sincerity. This does not mean that voters will necessarily vote sincerely, nor that they will behave the same when faced with Borda or approval voting systems. What we claim is that a voter who is sensitive to uncertainty and who therefore seeks a decision which is robust to uncertainty, will be inclined towards sincerity in both of these systems, depending on the voter's critical value. The empirical question is, In what situations are voters sufficiently uncertainty-averse to adopt a robust-satisficing strategy, even when manipulation is an option? Since Borda count and approval voting tend to be used in small settings such as committees, empirical studies in the laboratory could possibly provide meaningful insight.

5 Conclusion

Elections are always conducted in an uncertain environment. A voter cannot know the preferences of all other voters, nor can he know how these preferences will be aggregated into a vote. It is customary to assume that the voter forms utilities and subjective probabilities, based on the available data, of different outcomes of the elections. Based on these utilities and probabilities, the voter decides whether to vote at all, and if so, which alternative will yield the highest expected utility. However, the utilities and probabilities are prone to severe uncertainties, as the process of forming an estimate based on polls, commentaries, and so on, is far from exact. In this uncertain situation, a voter may prefer to satisfice his or her expected utility, rather than to maximize it.

In this paper we have defined the concept of robust-satisficing voting, which is distinguished from expected utility maximizing voting. We examined the consequences of robust-satisficing voting, compared to expected utility maximizing voting, under plurality voting with three candidates and no run-off. We have also studied n-candidate elections under plurality voting, as well as Borda-count and approval-vote systems.

We have shown that robust-satisficing voting under plurality elections is expected to be biased towards the sincere alternative, compared to expected utility maximizing voting. This result is coherent with empirical results, which show that strategic voting is not prevalent, even when Duverger's law predicts that it will be.

This paper suggests several directions for further investigation. Robust-satisficing has been invoked to explain the equity premium puzzle (Ben-Haim 2006) and the home-bias paradox (Ben-Haim and Jeske 2003) in financial economics. Robust-satisficing also provides a conceptual basis for understanding the foraging behavior of animals (Carmel and Ben-Haim 2005). These situations are characterized by competition for economic or physical survival under uncertainty. In all cases, the motivation for the robust-satisficing strategy is its survival-value. Survival does not require maximal utility; adequate outcomes (even when sub-optimal) are, by definition, sufficient. Choosing a policy that achieves adequate utility at the greatest horizon of uncertainty, the robust-satisficing agent enhances the chances of survival. The current paper explored the hypothesis that voters employ robust-satisficing rather than utility maximization. What are the implications of the survival-advantage of robust-satisficing for the process of social choice? As uncertainty increases, does the motivation for robust-satisficing become stronger, resulting in more sincere decisions? How should voting mechanisms be designed when robust-satisficing is used? What voting mechanisms encourage or discourage robust-satisficing behavior?

A Explicit Functional Form for Robustness Functions

In this appendix we will derive the explicit functional form for robustness functions under n-candidate plurality elections. The robustness functions of eq. (11)-(12) are a special case of these functions.

Consider the robustness function under the info-gap model defined in eq. (15):

$$\widehat{\alpha}(i, u_c) = \max\left\{\alpha : \left(\min_{p\in\mathcal{U}(\alpha,\widetilde{p})} U_i\right) \geq u_c\right\} \tag{17}$$

We will denote by $i \succ j$ the preference of candidate i over candidate j (*i.e.*, $u_i > u_j$). When $\alpha \leq 1$, the inner minimum of eq. (17) is:

$$\begin{aligned}
\min_{p\in\mathcal{U}(\alpha,\widetilde{p})} U_i &= \sum_{j\neq i} \min_{p\in\mathcal{U}(\alpha,\widetilde{p})} p_{ij}\Delta_{ij} \\
&= \sum_{j\prec i} \min_{p\in\mathcal{U}(\alpha,\widetilde{p})} p_{ij}\Delta_{ij} + \sum_{j\succ i} \min_{p\in\mathcal{U}'(\alpha,\widetilde{p})} p_{ij}\Delta_{ij} \\
&= \sum_{j\prec i} \left((1-\alpha)\widetilde{p}_{ij}\right)\Delta_{ij} + \sum_{j\succ i} \left((1+\alpha)\widetilde{p}_{ij}\right)\Delta_{ij} \\
&= \widetilde{U}_i - \alpha\left(\sum_{j\prec i}\widetilde{p}_{ij}\Delta_{ij} - \sum_{j\succ i}\widetilde{p}_{ij}\Delta_{ij}\right) \qquad (18)
\end{aligned}$$

We will define the following (monotonic) function of α:

$$\mu_i(\alpha) = \min_{p\in\mathcal{U}(\alpha,\widetilde{p})} U_i \tag{19}$$

Eq. (17) implies that $\widehat{\alpha}(i, u_c) = \mu_i^{-1}(u_c)$. Combining eqs. (17)-(19) yields:

$$\widehat{\alpha}(i, u_c) = \begin{cases} \frac{\widetilde{U}_i - u_c}{\sum_{j \prec i} \widetilde{p}_{ij} \Delta_{ij} - \sum_{j \succ i} \widetilde{p}_{ij} \Delta_{ij}} & , \text{ if } u_c < \widetilde{U}_i \\ 0 & , \text{ if } u_c \geq \widetilde{U}_i \end{cases} \tag{20}$$

Notice that the equation holds only for $\widehat{\alpha}(i, u_c) \leq 1$. However, this is always the case when $u_c \geq 0$.

B Proofs of Propositions

Proof of Proposition 1: Consider eqs. (11)-(12). Notice that for $u_c \leq 0$ we have that $\widehat{\alpha}(x, u_c) = \infty$. This means that for non-positive critical expected utility x is the most robust decision (or, if $\widehat{\alpha}(y, u_c) = \infty$, at least as robust). Therefore, we shall focus on positive critical expected utilities.

For $u_c = 0$, we have

$$\widehat{\alpha}(y, 0) = \frac{-\widetilde{p}_{xy} \Delta_{xy} + \widetilde{p}_{yz} \Delta_{yz}}{\widetilde{p}_{xy} \Delta_{xy} + \widetilde{p}_{yz} \Delta_{yz}} < 1 \tag{21}$$

When u_c approaches 0, we have

$$\lim_{u_c \to 0^+} \widehat{\alpha}(x, u_c) = 1 \tag{22}$$

This implies that even when $\widetilde{U}_x < \widetilde{U}_y$, that is, when there is an incentive for strategic voting, there is a range of values of the critical expected utility for which a robust-satisficing voter would prefer to vote sincerely. On the other hand, from the linear nature of the robustness functions over the positive critical expected utilities, when the expected utility maximizing voting is sincere (non-strategic), a robust-satisficing voter will always prefer the sincere voting. This notion is illustrated in Figure 1. ■

Proof of Proposition 2: Recall that the robustness of a decision under the info-gap model $\mathcal{U}'(\alpha, \widetilde{\Delta})$, defined by eq. (13), is:

$$\widehat{\alpha}'(i, u_c) = \max \left\{ \alpha \ : \ \left(\min_{p, \Delta \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} U_i \right) \geq u_c \right\} \tag{23}$$

First, we will claim that if a utility maximizing voter would have voted sincerely, so will a robust-satisficing voter.

Let $\alpha_{\max} = \max\left\{1, \frac{1}{h}\right\}$. Then from eq. (5) and eq. (13), when $\alpha \leq \alpha_{\max}$:

$$
\begin{aligned}
\min_{p \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} U_i &= \sum_{j \neq i} \min_{p \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} p_{ij} \Delta_{ij} \\
&= \sum_{j \prec i} \min_{p \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} p_{ij} \Delta_{ij} + \sum_{j \succ i} \min_{p \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} p_{ij} \Delta_{ij} \\
&= \sum_{j \prec i} ((1-\alpha)\widetilde{p}_{ij}) \left((1-h\alpha)\widetilde{\Delta}_{ij}\right) + \sum_{j \succ i} ((1+\alpha)\widetilde{p}_{ij}) \left((1+h\alpha)\widetilde{\Delta}_{ij}\right) \\
&= \sum_{j \prec i} ((1-(1+h)\alpha)\widetilde{p}_{ij})\, \widetilde{\Delta}_{ij} + \sum_{j \succ i} ((1+(1+h)\alpha)\widetilde{p}_{ij})\, \widetilde{\Delta}_{ij} + h\alpha^2 \sum_{j \neq i} \widetilde{p}_{ij} \widetilde{\Delta}_{ij} \\
&= \min_{p \in \mathcal{U}((1+h)\alpha, \widetilde{p})} U_i + h\alpha^2 \widetilde{U}_i \qquad (24)
\end{aligned}
$$

From Proposition 1 if follows that if the utility maximizing vote is x, then for any $u_c \geq 0$ it holds that $\widehat{\alpha}(x, u_c) \geq \widehat{\alpha}(y, u_c)$. This implies that for any $(1+h)\alpha \geq 0$,

$$
\min_{p \in \mathcal{U}((1+h)\alpha, \widetilde{p})} U_x \geq \min_{p \in \mathcal{U}((1+h)\alpha, \widetilde{p})} U_y \qquad (25)
$$

Since $\widetilde{U}_x \geq \widetilde{U}_y$ (the utility maximizing vote is x), we have:

$$
\min_{p, \Delta \in \mathcal{U}'(\alpha, \widetilde{p}, \widetilde{\Delta})} U_x \geq \min_{p, \Delta \in \mathcal{U}(\alpha, \widetilde{p}, \widetilde{\Delta})} U_y \qquad (26)
$$

and thus $\widehat{\alpha}'(x, u_c) \geq \widehat{\alpha}'(y, u_c)$.

Now, assume that $\widetilde{U}_y > \widetilde{U}_x$. From eq. (24) it follows that:

$$
\min_{p \in \mathcal{U}'(\alpha_{\max}, \widetilde{p}, \widetilde{\Delta})} U_x = 0 > \min_{p \in \mathcal{U}'(\alpha_{\max}, \widetilde{p}, \widetilde{\Delta})} U_y \qquad (27)
$$

Therefore, $\widehat{\alpha}'(x, 0) = \alpha_{\max} > \widehat{\alpha}'(y, 0)$. But since $\widetilde{U}_y > \widetilde{U}_x$, then $\widehat{\alpha}'(x, U_x) = 0 < \widehat{\alpha}'(y, U_x)$. This means that there is a reversal of preference, like the one described in Figure 1b. Thus, for some (positive) values of u_c, the robust-satisficing vote is x, although the utility maximizing vote is y. ■

Proof of Proposition 3: Consider eq. (20). Notice that $\sum_{j \succ i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j} = 0$, while for any $i \neq i_1$, $\sum_{j \succ i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j} \leq 0$. Therefore:

$$
\widehat{\alpha}(i_1, 0) = \frac{\widetilde{U}_{i_1}}{\sum_{j \prec i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j} - \sum_{j \succ i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j}} = \frac{\sum_{j \prec i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j} + \sum_{j \succ i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j}}{\sum_{j \prec i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j} - \sum_{j \succ i_1} \widetilde{p}_{i_1 j} \Delta_{i_1 j}} = 1 \qquad (28)
$$

while for any $i \neq i_1$:

$$\widehat{\alpha}(i,0) = \frac{\widetilde{U}_i}{\sum_{j \prec i} \widetilde{p}_{ij}\Delta_{ij} - \sum_{j \succ i} \widetilde{p}_{ij}\Delta_{ij}} = \frac{\sum_{j \prec i} \widetilde{p}_{ij}\Delta_{ij} + \sum_{j \succ i} \widetilde{p}_{ij}\Delta_{ij}}{\sum_{j \prec i} \widetilde{p}_{ij}\Delta_{ij} - \sum_{j \succ i} \widetilde{p}_{ij}\Delta_{ij}} \leq 1 \tag{29}$$

Thus, $\widehat{\alpha}(i_1,0) \geq \widehat{\alpha}(i,0)$ for any i.

Since $\widehat{\alpha}(i,u_c)$ is linear in u_c, if $\widetilde{U}_{i_1} \geq \widetilde{U}_i$ then for any $u_c \geq 0$ it holds that $\widehat{\alpha}(i_1,u_c) \geq \widehat{\alpha}(i,u_c)$ (recall that $\widehat{\alpha}(i_1,\widetilde{U}_{i_1}) = \widehat{\alpha}(i,\widetilde{U}_i) = 0$). This is the situation described in Figure 1a.

However, if $\widetilde{U}_{i_1} < \widetilde{U}_i$ then there is a reversal of preference, as described in Figure 1b. This means that a robust-satisficing voter may vote for i_1 although the utility maximizing vote is $i \neq i_1$. ■

Lemma 7 *Bias towards sincerity is a transitive property.*
Given *three candidates $a \succ b \succ c, \widetilde{U}_a, \widetilde{U}_b, \widetilde{U}_c > 0$, and the robustness functions of the candidates are linear in u_c,* **if** *when comparing a and b there is a bias towards a, and when comparing b and c there is a bias towards b,* **then** *when comparing a and c there is a bias towards a.*

Proof of Lemma 7: If $\widetilde{U}_a > \widetilde{U}_b$, then for any $u_c \geq 0$ it holds that $\widehat{\alpha}(a,u_c) \geq \widehat{\alpha}(b,u_c)$. If $\widetilde{U}_a < \widetilde{U}_b$, then there exists $u_c \geq 0$ for which $\widehat{\alpha}(a,u_c) \geq \widehat{\alpha}(b,u_c)$. As demonstrated by Figure 1, both cases imply $\widehat{\alpha}(a,0) \geq \widehat{\alpha}(b,0)$. Similarly, we may conclude that $\widehat{\alpha}(b,0) \geq \widehat{\alpha}(c,0)$, and therefore $\widehat{\alpha}(a,0) \geq \widehat{\alpha}(c,0)$.

From Figure 1 it is apparent that if $\widehat{\alpha}(a,0) \geq \widehat{\alpha}(c,0)$ then a bias towards sincerity exists. ■

Proof of Proposition 4: Instead of proving the proposition for any i and j, we will prove that the proposition holds for i_k and i_l, where $i_1 \succ \ldots i_k \succ i_l \succ \ldots i_n$.

Consider eq. (20). For i_k and i_l it can be rephrased as follows:

$$
\begin{aligned}
\widehat{\alpha}(i_k,0) &= \frac{\widetilde{U}_{i_k}}{\sum_{j\prec i_k}\widetilde{p}_{i_kj}\Delta_{i_kj}-\sum_{j\succ i_k}\widetilde{p}_{i_kj}\Delta_{i_kj}} \\
&= \frac{\sum_{j\prec i_l}\widetilde{p}_{i_kj}\Delta_{i_kj}+\widetilde{p}_{i_ki_l}\Delta_{i_ki_l}+\sum_{j\succ i_k}\widetilde{p}_{i_kj}\Delta_{i_kj}}{\sum_{j\prec i_l}\widetilde{p}_{i_kj}\Delta_{i_kj}+\widetilde{p}_{i_ki_l}\Delta_{i_ki_l}-\sum_{j\succ i_k}\widetilde{p}_{i_kj}\Delta_{i_kj}} \\
&= \frac{\overbrace{\sum_{j\prec i_l}\widetilde{q}_j\Delta_{i_kj}}^{\xi_1}+\widetilde{q}_{i_l}\Delta_{i_ki_l}-\overbrace{\left(-\sum_{j\succ i_k}\widetilde{q}_j\Delta_{i_kj}\right)}^{\xi_2}}{\sum_{j\prec i_l}\widetilde{q}_j\Delta_{i_kj}+\widetilde{q}_{i_l}\Delta_{i_ki_l}+\left(-\sum_{j\succ i_k}\widetilde{q}_j\Delta_{i_kj}\right)} \\
&= \frac{\xi_1+\widetilde{q}_{i_l}\Delta_{i_ki_l}-\xi_2}{\xi_1+\widetilde{q}_{i_l}\Delta_{i_ki_l}+\xi_2} \qquad (30)\\
\widehat{\alpha}(i_l,0) &= \frac{\widetilde{U}_{i_l}}{\sum_{j\prec i_l}\widetilde{p}_{i_lj}\Delta_{i_lj}-\sum_{j\succ i_l}\widetilde{p}_{i_lj}\Delta_{i_lj}} \\
&= \frac{\sum_{j\prec i_l}\widetilde{p}_{i_lj}\left(\Delta_{i_kj}-\Delta_{i_ki_l}\right)-\widetilde{p}_{i_li_k}\Delta_{i_ki_l}+\sum_{j\succ i_k}\widetilde{p}_{i_lj}\left(\Delta_{i_kj}-\Delta_{i_ki_l}\right)}{\sum_{j\prec i_l}\widetilde{p}_{i_lj}\left(\Delta_{i_kj}-\Delta_{i_ki_l}\right)+\widetilde{p}_{i_li_k}\Delta_{i_ki_l}-\sum_{j\succ i_k}\widetilde{p}_{i_lj}\left(\Delta_{i_kj}-\Delta_{i_ki_l}\right)} \\
&= \frac{\sum_{j\prec i_l}\widetilde{q}_j\Delta_{i_kj}-\overbrace{\sum_{j\prec i_l}\widetilde{q}_j}^{\xi_3}\Delta_{i_ki_l}-\widetilde{q}_{i_k}\Delta_{i_ki_l}-\left(-\sum_{j\succ i_k}\widetilde{q}_j\Delta_{i_kj}\right)-\overbrace{\sum_{j\succ i_k}\widetilde{q}_j}^{\xi_4}\Delta_{i_ki_l}}{\sum_{j\prec i_l}\widetilde{q}_j\Delta_{i_kj}-\sum_{j\prec i_l}\widetilde{q}_j\Delta_{i_ki_l}+\widetilde{q}_{i_k}\Delta_{i_ki_l}+\left(-\sum_{j\succ i_k}\widetilde{q}_j\Delta_{i_kj}\right)+\sum_{j\succ i_k}\widetilde{q}_j\Delta_{i_ki_l}} \\
&= \frac{\xi_1-\left(\xi_3+\widetilde{q}_{i_k}+\xi_4\right)\Delta_{i_ki_l}-\xi_2}{\xi_1+\left(-\xi_3+\widetilde{q}_{i_k}+\xi_4\right)\Delta_{i_ki_l}+\xi_2} \qquad (31)
\end{aligned}
$$

Notice that all the arguments within the functions, that is $\xi_1,\ldots,\xi_4,\widetilde{q}_{i_k},\widetilde{q}_{i_l},\Delta_{i_ki_l}$, are non-negative.

In the proof of Proposition 3 we have seen that $\widehat{\alpha}(i_1,0)\geq\widehat{\alpha}(i,0)$ for any i. Could it be that $\widehat{\alpha}(i_k,0)\leq\widehat{\alpha}(i_l,0)$? Using eqs (30)-(31), this is equivalent to:

$$\frac{\xi_1+\widetilde{q}_{i_l}\Delta_{i_ki_l}-\xi_2}{\xi_1+\widetilde{q}_{i_l}\Delta_{i_ki_l}+\xi_2}\leq\frac{\xi_1-\left(\xi_3+\widetilde{q}_{i_k}+\xi_4\right)\Delta_{i_ki_l}-\xi_2}{\xi_1+\left(-\xi_3+\widetilde{q}_{i_k}+\xi_4\right)\Delta_{i_ki_l}+\xi_2} \qquad (32)$$

From eq. (20) it is apparent that both denominators are positive. Thus, the inequality implies:

$$0\leq\xi_1\leq-\frac{\xi_2\xi_3+\widetilde{q}_{i_l}\widetilde{q}_{i_k}\Delta_{i_ki_l}+\widetilde{q}_{i_l}\Delta_{i_ki_l}\xi_4}{\xi_4+\widetilde{q}_{i_k}} \qquad (33)$$

This is only possible when $\widetilde{q}_{i_k} = \widetilde{q}_{i_l} = 0$, which implies $\widetilde{U}_{i_k} = \widetilde{U}_{i_l} = \widehat{\alpha}(i_k, u_c) = \widehat{\alpha}(i_l, u_c) = 0$ for any $u_c > 0$, making both i_k and i_l irrelevant for both utility maximizing and robust-satisficing voters.

We have seen that if both i_k and i_l are legitimate possibilities, then $\widehat{\alpha}(i_k, 0) \geq \widehat{\alpha}(i_l, 0)$. This implies, as shown previously, that when $\widetilde{U}_{i_k} > \widetilde{U}_{i_l}$ we will have $\widehat{\alpha}(i_k, u_c) \geq \widehat{\alpha}(i_l, u_c)$ for any $u_c \geq 0$, while if $\widetilde{U}_{i_k} < \widetilde{U}_{i_l}$ there exists a reversal of preferences. This reversal of preferences constitutes a bias towards the more sincere vote, i_k. ■

Proof of Corollary 5: If b is more sincere than c, then b can be reached from c by a series of steps in which a single point is "reallocated" from candidate i_l to candidate i_k, where $i_k \succ i_l$. Although the intermediate scoring is not a valid Borda scoring, it is still possible to show that in each such step there is bias towards the more sincere score.

When examining two scores, different in a single point, we treat this case as equivalent to plurality voting: the voter has to decide to whom will the "excess" vote go. Since this case is equivalent to plurality voting Proposition 4 applies, and hence there is a bias towards the more sincere score, in which the excess vote is given to the more preferred candidate. ■

Proof of Corollary 6: Assume without loss of generality that $i_1 \succ \cdots \succ i_n$. Let k be the minimal value such that $c_k = 0$, and let l be the maximal value such that $c_l = 1$. Since c is a strategic vote we have $k < l$.

We will define d as follows:

$$d_j = \begin{cases} c_j & \text{, if } j \neq k, l \\ 0 & \text{, if } j = l \\ 1 & \text{, if } j = k \end{cases} \quad (34)$$

Notice that d is "more sincere" than c, and that after a finite number of such steps it is possible to reach a sincere vote b. We will show that between c and d, under robust-satisficing voting there is a bias towards d.

Let A_c denote the set of approved candidates under vote c. That is, $c_i = 1$ if and only if $i \in A_c$. We will define A_d similarly, and let $A = A_c \cap A_d$.

Now assume that we have already decided whether or not to approve all of the candidates, except for candidates i_k and i_l. That is, we haven't decided whether we prefer c or d. We can treat this final choice as a plurality voting, when we choose between candidates i_k and i_l. In this plurality voting, the expected utility from voting to candidate $i \in \{i_k, i_l\}$ is:

$$U_i = \sum_{j \neq i, j \notin A} p_{ij} \Delta_{ij} \quad (35)$$

This is equivalent to a plurality voting between $\{i_1, \ldots, i_n\} \setminus A$. Proposition 3 implies that in this case there is a bias towards candidate i_k, which is the most preferred candidate. Thus, between vote c and vote d there is a bias towards d. ■

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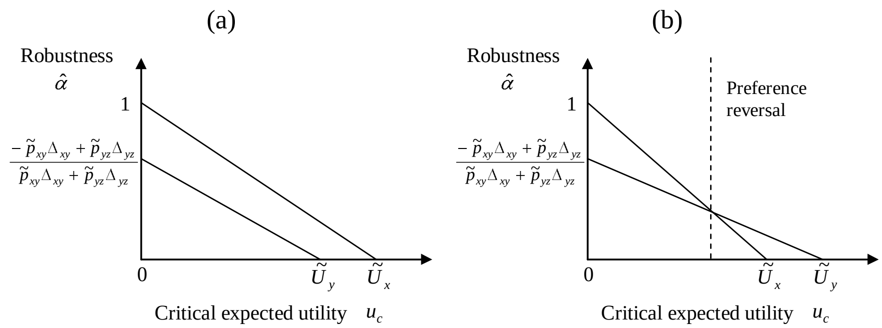

Figure 1: Robustness Curves for the Two Voting Alternatives. Figure 1a illustrates a case where the expected utility maximizing voting is sincere. In this case, the robust-satisficing voting is also sincere. Figure 1b illustrates a case where the expected utility maximizing voting is strategic (differs from the sincere voting). In this case, a robust-satisficing voter may vote similarly to a sincere voter.