Robust Reliability of Projects with Activity-Duration Uncertainty

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Abstract: Today's projects suffer from high uncertainty, great demand for accelerated speed, and severe scarcity of attention of the project manager. At the same time, the schedules of today's projects are expected to be more reliable than ever. This paper presents a new concept for improving the reliability of a project schedule suffering from uncertainty in the duration of its activities. The paper shows that the technique for applying the new concept requires minimal information, incorporates subjective information, is simple to use, and assists in the preparation of project schedules at a desirable level of reliability. Specific examples demonstrate the use of the technique for: (1) calculating the reliability of the project schedule; (2) enhancing the reliability of the project schedule; (3) reducing project duration without diminishing its reliability; and (4) examining how overlapping of project activities affects its reliability.

INTRODUCTION

Coping with uncertainty in the duration of project activities has received much research attention since the early days of modern project scheduling tools (Gurbbs 1962; Martin 1965). While this research interest has intensified in recent years (Frizelle 1993; Gong and Hugsted 1993; Hulett 1995; Huseby and Skogen 1992; Loterapong and Moselhi 1996; Simister 1994), it has not yielded a widespread use of sophisticated techniques developed for coping with uncertainty. Shonberger (1981) was able to demonstrate that "projects will always be late—relative to the deterministic critical path," but he concluded nevertheless that: "It does not seem particularly useful for the project manager to stand the expense of simulating the project network."Recent research (Simister 1994) that examined the actual use of the available techniques for analyzing project uncertainty found that the technique mostly used by practicing project managers is still the simple checklist.

Project managers in today's dynamic environment deserve more than a technique that involves only a simple checklist. In this paper a new concept concerning the assessment of the reliability of a project schedule, and a new technique for applying this concept is presented. The technique requires only minimal effort, yet provides the project manager with vital information for coping with uncertainty.

In the remainder of this section first the unique characteristics of the problem of project planning and scheduling in today's dynamic environment are presented, followed by the characteristics of the solution proposed by this paper.

Characteristics of Problem

Uncertainty

Laufer and Tucker (1988) concluded that uncertainty is not an exceptional state in the otherwise predictable process of construction work. In fact it is a permanent feature in the realm of construction, obviously resulting from conditions prevailing at the construction site and its environment. Recent studies, however, have shown that uncertainty originates at much higher hierarchy levels and is rooted in a much earlier stage of the project life. Laufer (1991) and Howell et al. (1993) found that in many projects construction starts at a high level of uncertainty, due to the incompleteness and instability of project objectives emanating from the project owner. Laufer (1996) showed that because today's world is quite dynamic and projects must respond to a more compressed schedule, uncertainty has become one of the major factors that influence a project's performance and ultimate success.

Speed

Project speed has recently become a competitive requirement. In today's business the winner is the competitor who consistently, reliably, and profitably provides the greatest value to the customer, and does it prior to the other competitors (Stalk and Hout 1990; Peters 1989; Meyer 1993). There are several reasons why speed has become a competitive requirement: (1) Dramatic increase in global competition; (2) accelerated pace of technological development; and (3) both market share and profit margins are increased by being first in the market. For example, a widely-cited financial model developed by McKinsey & Co. indicates that a product that is on budget but six months late to market, misses out on one-third of the potential profit over its lifetime (Smith and Reinersten 1991). Whether the assumptions of the model are valid for a particular project, or whether that project is at all related to a new product, is irrelevant nowadays. "Speed fever" is contagious and customers everywhere expect their projects to be completed with ever-increasing swiftness.

Scarcity of Attention

There is more than enough evidence to indicate that managers do not have the time necessary for project planning (Laufer and Tucker 1987). Research shows that managers' activities are typified by brevity, variety, and fragmentation. Half of the activities of American executives were found to last less than 9 min, and only 10% of those activities exceeded 1 h. In a similar study made in Britain, it was found that middle and top managers were able to work without interruption for half an hour or more only once every 2 d (Mintzberg 1973; Steward 1967; Dinsmore 1982). Empirical studies portray the manager's life as leaving little time for reflection and analysis amid the pressure of short-term, interrupted, and somewhat chaotic activities. Furthermore, the manager's shortage of time is more severe under conditions of high uncertainty, since more information must be processed during execution. Not surprisingly, recent advances in information technology do not ameliorate the problem. Time pressures are further increased and probably amplified by telefaxes, cellular phones, and electronic mail systems. In general, computers seem to have done more to increase the information load than to reduce it.

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The preceding discussion indicates that the three factors: (1) uncertainty; (2) speed; and (3) scarcity of attention are very much interrelated. The greater the uncertainty, the greater the scarcity of attention; whereas the greater the speed, the greater the uncertainty and the scarcity of attention.

Characteristics of Required Solution

Minimal Information

To be useful, the proposed solution must not require from the user any more than very minimal information about the uncertain phenomena.

Subjective Information

At the same time, the proposed method should be capable of incorporating and benefiting from specific, subjective information the user may possess. This is in sharp contrast with most common techniques proposed in the literature where the underlying assumption concerning uncertainty is that everything is unknown to the same degree. That is, when schedule risk analysis is performed using a simulation technique, the statistical technique typically employs the same probability estimate for all project activities. In most cases the statistical distribution used is not based on particular information the user has about the current project. The usual recommendation is to use an "average" convenient distribution (Hulett 1995). Laufer (1996), however, reports that successful project managers demonstrate that they are able to assess subjectively the degree of uncertainty of some of the project activities, and that this ability to discriminate between quite-certain, uncertain, and very uncertain activities, is a key to their success.

Simplicity

In a dynamic situation the method will have to be applied not only prior to the beginning of construction but rather several times throughout construction (Laufer et al. 1994; Couillard 1995). To make sure that busy project managers will apply the method during construction as well, it must be simple, easy to understand, and its implementation should require limited effort for collecting and processing of data, and for interpreting the results of the analysis.

Enhancement and Assessment of Reliability

The paradox of our era is that as managers have to face more constraints, they are also expected to achieve better project performance. In our case, the more the project suffers from high uncertainty and accelerated speed, the higher the expected reliability of the project schedule. This is a direct implication of the overriding rule of making business today. That is, provide top quality, as perceived by the customer; and a reliable schedule is perceived by customers to be a key component of quality. To achieve high reliability, project managers should be equipped with a concept and technique that enable them to focus on reliability of project results, to assess it, and to examine alternative ways of improving it with minimum additional resources.

The following section describes the concept of information-gap uncertainty and its quantification with convex models of uncertainty. The main results of the paper—the robust reliability analysis of a project with activity-duration uncertainty—are then presented and illustrated with several examples.

INFORMATION-GAP UNCERTAINTY AND CONVEX MODELS

One useful classification of uncertain phenomena is to distinguish between "structured" and "unstructured" uncertainty. An example of a structured uncertainty is the ordinary variation, from year to year, of the temperature on a given date. Extensive data are available from which a probability density function can be constructed for predicting mean, standard deviation, and other statistical properties of the temperature. An unstructured uncertainty, on the other hand, may be a surprise, an unexpected turn of events, an event different from all or nearly all previous experience. Meteorological examples would be rain in the dry season, snow in the summer, etc. Or, unstructured uncertainty may arise from severe lack of information about mundane contingencies.

The distinction between structured and unstructured uncertainties arises from the varying extent of our knowledge. Given sufficient experience, what was previously a surprise becomes only a rare but nonetheless foreseeable event. In other words, a more precise formulation of unstructured uncertainty is: uncertainty associated with severe lack of information. Unstructured uncertainty is a substantial information-gap between what we do know and what we need to know to perform optimally. This paper will be concerned with information-gap uncertainty and the design of project schedules for dealing with this uncertainty.

When dealing with a severe lack of information one must be very parsimonious with the information available. This has two main implications. First, unverifiable assumptions must be avoided as much as possible. In particular, probability densities will be unable to be adopted, since it is the rare events—surprises—that dominate our concern, and there is no way to choose the tails of the distribution. Second, strong statistical assertions cannot be made, as though facing very structured and well documented uncertainty. Rather than "probabilistic" reliability, "robustness" will be adopted as the measure of reliability.

Robust reliability is defined as the amount of uncertainty consistent with no-failure of the project. A project schedule is reliable if it is robust or immune to large unknown variations that arise during implementation. On the other hand, a project is unreliable if it is fragile or vulnerable to uncertainty. The degree of robustness is a measure of the project reliability. Precise quantitative meaning to this idea will be given. In particular, a precise formulation of the amount of uncertainty will be presented. This will be based on convex models of uncertainty rather than on probabilistic models, because of dealing with unstructured rather than structured uncertainty.

The idea of robust reliability has proven fruitful in the analysis of mechanical systems subject to severe lack of information (Ben-Haim 1996, 1997) and in this paper it will be shown that the same ideas are applicable to project scheduling.

Before the analysis of project reliability begins, convex models of uncertainty are briefly discussed. Extensive technical discussion can be found elsewhere (Ben-Haim 1985, 1996; Ben-Haim and Elishakoff 1990). A nontechnical discussion of convex models is also available (Ben-Haim 1994).

Consider an example: uncertain variation in the cost $c(t)$ of raw material as a function of time during implementation of the project. The normal or anticipated temporal variation of the cost is $\tilde{c}(t)$, which is a known function. The actual cost $c(t)$ deviates by an unknown amount from the nominal cost $\tilde{c}(t)$. However, it is expected that the cost variation will be greater as time goes on: the belief in $\tilde{c}(t)$ is much less for long duration than for short duration. A function $\psi(t)$ was chosen that equals 1 at $t = 0$ (start of the project) and increases in time to express feelings of the increasing range of cost variation. To summarize, it is unknown how likely one cost function is when compared with another. (Likelihood is characteristic of structured uncertainty, which could be represented with a probabilistic model.) However, a nominal cost function is known, and there is a rough idea of how the range of variation should increase over time.

This information is readily quantified in a convex model of uncertainty. Consider the set of all cost functions $c(t)$, whose deviation from the nominal function $\tilde{c}(t)$ is bounded by $\alpha\psi(t)$

$$\mathscr{C}(\alpha, \tilde{c}) = \{c(t): |c(t) - \tilde{c}(t)| \le \alpha\psi(t)\} \tag{1}$$

$\mathscr{C}(\alpha, \tilde{c})$ is a set of functions: it contains all cost functions consistent with our prior information, where α is the "uncertainty parameter," expressing the (unknown) degree of variation of the cost function; and $\psi(t)$ defines a time-varying envelope within which the cost function varies.

This convex model, $\mathscr{C}(\alpha,\tilde{c})$, is a family of nested convex sets for $\alpha \ge 0$. This means that $\mathscr{C}(\alpha, \tilde{c}) \subseteq \mathscr{C}(\beta, \tilde{c})$ if $\alpha \le \beta$. Uncertainty is expressed at two levels by this convex model. For fixed α, the set $\mathscr{C}(\alpha, \tilde{c})$ represents a degree of uncertain variability of the cost function $c(t)$. The greater the value of α, the greater the variation, so α, the uncertainty parameter, expresses the information gap between what is known [$\tilde{c}(t)$ in the preceding example] and what needs to be known for an ideal solution [the exact function $c(t)$]. The value of α is usually unknown, which constitutes the second level of uncertainty.

Each set in the family of uncertainty-sets defined in (1) is in fact a convex set. The convexity of the sets has not been assumed; the convexity simply arises as a by-product of how the partial information is quantified. Convex uncertainty-sets are frequently encountered, and attractive analytical simplifications result from the convexity; hence the name: convex model.

In the convex model of (1), each set in the family is defined as the collection of all functions consistent with the prior information, up to uncertainty α. This is characteristic of how convex models are formulated in general. In this way, the convex model is constructed with very parsimonious use of information.

RELIABILITY OF ACTIVITY-PATH STRUCTURE OF PROJECT

In this section the concept of robust reliability for evaluating the activity-path structure of a project-plan is applied. The project is made up of a number of activities, whose durations are uncertain. The project as a whole must be completed within a specified time. The "plan" is an organization of the activities into the sequences (activity paths) in which they will be performed. The end result of the analysis is an assessment of how robust the plan is to the uncertainties in the activity durations. Alternative plans in terms of their robustnesses were quantitatively compared. For instance, the utility can be determined, in terms of added robustness to uncertainty, of employing alternative technologies that allow different activity sequencing.

Fig. 1 shows the plan of a hypothetical 16-activity project: a flowchart of the sequence of execution of the activities. The project is organized into five activity paths:

Path 1: $1 \to 2 \to 3 \to 4 \to 16$.
Path 2: $1 \to 5 \to 6 \to 3 \to 4 \to 16$.
Path 3: $1 \to 5 \to 6 \to 7 \to 8 \to 16$.
Path 4: $1 \to 5 \to 6 \to 9 \to 10 \to 11 \to 16$.
Path 5: $1 \to 12 \to 13 \to 14 \to 15 \to 16$.

It is noted that in several places a given activity appears in several paths. This is a result of the fact that the paths interlace.

The project is completed successfully if all of the activities are completed within the allowed duration, T_{cr}. The questions we ask are: how robust is the successful completion of this project to the uncertain duration of the activities? Can the robustness be increased by a different organization of the activities into activity paths? And if so, by how much?

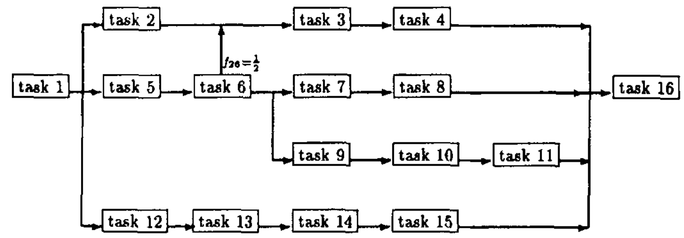

FIG. 1. Sixteen-Activity Project Schedule for Example 1

This is now formulated generically, and a solution is presented that is based on convex models of uncertainty and robust reliability. The reliability analysis involves three components: The "dynamic model" expresses the relation between the activity-pathing and the project duration. The "failure criterion" states the conditions under which the project does not succeed. Finally, the "uncertainty model" quantifies the uncertainties that accompany the project; in this case the uncertain duration of the activities.

The dynamic model is formulated first, which tells how long the project takes. The actual duration of the nth activity is denoted t_n, for $n = 1, \ldots, N$, where N is the number of activities. The vector of activity times is $\mathbf{t} = (t_1, \ldots, t_N)^T$. In the example in Fig. 1, $N = 16$.

The activities are organized into M activity paths. In Fig. 1, $M = 5$ as explained earlier. f_{mn} is the fractional participation of activity n in path m. This means that in path m the activity following the nth starts when activity n is a fraction f_{mn} complete. F is the activity path participation matrix. This is the $\mathbf{M} \times \mathbf{N}$ matrix of numbers f_{mn} between 0 and 1. For instance, the participation matrix for the project plan in Fig. 1 is

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \end{pmatrix} \tag{2}$$

The mth row represents the mth activity path of the flow chart of Fig. 1. For example, the value 1/2 in the second row represents the fact that, in the second activity path, activity 3 begins when activity 6 is one-half done.

The duration of the mth activity path is the sum of durations of the activities in that path, weighted by their fractional participation times:

$$c_m = \sum_{n=1}^{N} f_{mn} t_n, \quad m = 1, \ldots, M \tag{3}$$

For instance, the duration of the second path is $c_2 = 1 \cdot t_1 + 1 \cdot t_3 + 1 \cdot t_4 + 1 \cdot t_5 + 1/2 \cdot t_6 + 1 \cdot t_{16}$.

Eq. (3) can be expressed as a vector relation between path durations and activity durations:

$$\mathbf{c} = F\mathbf{t} \tag{4}$$

where $\mathbf{c}$ = vector of path durations.

The dynamic model is the duration of the longest path:

$$T = \|\mathbf{c}\| = \max_{1 \le m \le M} |c_m| = \max_{1 \le m \le M} \left| \sum_{n=1}^{N} f_{mn} t_n \right| \tag{5}$$

Note that $\|\mathbf{c}\|$ is in fact a vector norm, sometimes called the "zero norm."

The failure criterion states that the project fails if the duration of the longest path exceeds a critical value. Failure is

$$T > T_{cr} \tag{6}$$

Very little may be known about the variation in the activity durations. That is, the gap between what is known about the durations, and what needs to be known to make a perfect plan, may be quite substantial. This information gap results from incomplete familiarity with the conditions under which the project will be executed, and from the fact that surprises—unexpected occurrences—are bound to arise. This information gap is a form of uncertainty that is usefully represented by a convex model.

Let us suppose that there is some prior knowledge about the typical or nominal duration of each activity, but that very little is known about how much the actual duration will deviate from the nominal value. Also, there may be some rough information about the relative variability of the different activities. Let $\bar{t}_n$ denote the nominal duration of the n activity, for $n = 1, \ldots, N$. The coefficients $w_1, \ldots, w_N$ are positive numbers expressing the relative variability of the activities. If there is no prior information about the relative variability of the activities, then all the w_n will equal unity. If the nth activity tends to vary more than the others, then its uncertainty coefficient, w_n, will exceed 1. Conversely, activities that tend to vary less than most will have w_n less than 1.

A simple uncertainty model based on this information states that each activity duration may deviate by an unknown fraction of its nominal value. Consider the following "uniform-bound" convex model:

$$\mathcal{T}(\alpha) = \left\{ \mathbf{t} : \frac{|t_n - \bar{t}_n|}{\bar{t}_n} \le w_n\alpha, \quad n = 1, \ldots, N \right\} \qquad (7)$$

$\mathcal{T}(\alpha)$ is an infinite set of values of the vector $\mathbf{t}$ of activity durations. Each element t_n of the vector $\mathbf{t}$, representing the duration of the nth activity, varies within the interval

$$\bar{t}_n - w_n\bar{t}_n\alpha \le t_n \le \bar{t}_n + w_n\bar{t}_n\alpha \qquad (8)$$

In other words, $\mathcal{T}(\alpha)$ is the set of $\mathbf{t}$ vectors whose elements t_n vary from their nominal values by no more than a fraction $w_n\alpha$. If the w_n are all unity, then the maximum fractional variations of the activity durations are all equal to α. Alternatively, if each w_n equals $1/\bar{t}_n$, then α becomes the maximum absolute variation of the activity durations. Other choices of the w_n can express other fragmentary information about the relative variability of the activity durations. In example 1 a particular choice of w_n is considered. So, $\mathcal{T}(\alpha)$ is the set of all activity-time vectors whose uncertainty is less than α, which is called the uncertainty parameter. While the nominal durations $\bar{t}_n$ are known, the value of α is not known, which expresses the uncertainty in the activity durations. So α is the independent variable and $\mathcal{T}(\alpha)$ is a set-valued function of α. In other words, $\mathcal{T}(\alpha)$ is a family of nested sets, for $\alpha \ge 0$, which simply means that the range of uncertain variation of the activity durations increases with α. This is expressed by the "nesting" of the sets, which means that if $\alpha < \beta$, then $\mathcal{T}(\alpha) \subset \mathcal{T}(\beta)$. Since these sets are in fact convex, this is precisely a convex model.

The "robust reliability" is the greatest value of the uncertainty parameter, α, which is consistent with no-failure of the project. This is evaluated as follows. First a set of acceptable α-values is defined: those values that do not allow failure. Consider the set

$$\mathcal{A}(T_{cr}) = \left\{ \alpha : \max_{1\le m\le M} \left| \sum_{n=1}^{N} f_{mn}t_n \right| \le T_{cr} \quad \text{for all} \quad t \in \mathcal{T}(\alpha) \right\} \qquad (9)$$

In light of (5) and (6), it is seen that $\mathcal{A}(T_{cr})$ is the set of α-values for which all activity-duration vectors t in $\mathcal{T}(\alpha)$ lead to successful project completion. The robust reliability of the project plan is the greatest acceptable value of the uncertainty parameter

$$\hat{\alpha} = \max_{\alpha\in\mathcal{A}(T_{cr})} \alpha \qquad (10)$$

$\hat{\alpha}$ = maximum of the set of α-values for which the greatest path-duration is acceptable (no failure) for all activity times in the uncertainty-set $\mathcal{T}(\alpha)$. It is seen in this expression for the robust reliability that all three components are combined: the dynamic model, the failure criterion, and the uncertainty model.

An explicit algebraic expression is now developed for the robust reliability, followed by several examples. The greatest duration of the mth activity path occurs when each activity runs maximally overtime. From (8) it is seen that the greatest duration of activity n is $\bar{t}_n + w_n\bar{t}_n\alpha$ where, of course, the value of α is unknown. So, the maximum duration of the mth path is

$$\max_{t\in\mathcal{T}(\alpha)} c_m = \max_{t\in\mathcal{T}(\alpha)} \sum_{n=1}^{N} f_{mn}t_n = \sum_{n=1}^{N} f_{mn}(\bar{t}_n + w_n\bar{t}_n\alpha)$$

$$= \underbrace{\sum_{n=1}^{N} f_{mn}\bar{t}_n}_{\bar{c}_m} + \alpha \underbrace{\sum_{n=1}^{N} f_{mn}w_n\bar{t}_n}_{f_m} \qquad (11)$$

$$\max_{t\in\mathcal{T}(\alpha)} c_m = \bar{c}_m + \alpha f_m \qquad (12)$$

where $\bar{c}_m$ and f_m are defined in (11). The absolute value signs have been dropped because all of the terms are nonnegative.

The robustness is found by solving the following relation for $\hat{\alpha}$

$$\max_{1\le m\le M} [\bar{c}_m + \hat{\alpha} f_m] = T_{cr} \qquad (13)$$

To do this, define the quantities

$$\alpha_m = \frac{T_{cr} - \bar{c}_m}{f_m}, \quad m = 1, \ldots, M \qquad (14)$$

α_m is the greatest acceptable uncertainty that the mth activity path can tolerate without violating the failure criterion [(6)]. In other words, α_m is the robustness of path m. It is required that all paths not fail, so the logical structure of this problem is similar to a serial network of subunits, where each subunit is essential for operation of the network. The robustness of the entire project is the robustness of the weakest path. That is, the robustness is the least of these α_m

$$\hat{\alpha} = \min_{1\le m\le M} \frac{T_{cr} - \bar{c}_m}{f_m} \qquad (15)$$

Example 1: Reliability of Project Schedule

Consider the 16-activity project-plan whose structure is shown in Fig. 1 and whose participation matrix is presented in (2). The nominal times $\bar{t}_n$ and uncertainty weights w_n are recorded in Table 1. The uncertainty weights w_n for activities 2, 3, 4, and 6 all equal 2 since prior information indicates that the duration of these activities tends to deviate up to twice as much as the other activities. The uncertainty weight for activity 9 is $w_9 = 0.5$ since this activity tends to deviate less than the others.

The reliability of the network will change as a function of

TABLE 1. Nominal Durations and Uncertainty Weights for Example 1

(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)	(15)	(16)	(17)
n	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
$\bar{t}_n$	1	4	6	3	2	3	5	4	4	2	1	2	1	3	1	2
w_n	1	2	2	2	1	2	1	1	0.5	1	1	1	1	1	1	1

TABLE 2. Path Robustnesses with Various Allotted Activity Durations for Example 1

T_{cr} (1)	α_1 (2)	α_2 (3)	α_3 (4)	α_4 (5)	α_5 (6)
17	0.035	0.058	**0.00**	0.13	0.70
19	**0.10**	0.14	**0.10**	0.25	0.90
21	**0.17**	0.21	0.20	0.38	1.10

the total time allotted for project completion. Table 2 shows the robustnesses of the five paths, α_m calculated according to (14), for several values of the allotted total activity duration, T_{cr}. The boldfaced number in each row is the lowest path-robustness which, according to (15), is the robustness of the entire project schedule for that value of T_{cr}. When the allotted duration is T_{cr} = 17 time units, the third path has zero robustness (α_3 = 0). This occurs because the nominal duration of activity-path 3 (the sum of its $\tilde{t}_n$) precisely equals 17 time units. Consequently, even the slightest time overrun results in a project overrun, according to condition (6), so this path has zero immunity to activity-duration uncertainty.

All of the path-robustnesses increase as the allotted project duration T_{cr} increases, as expected from (14). At T_{cr} = 19 the project robustness is $\hat{\alpha} = \hat{\alpha}_1 = \hat{\alpha}_3 = 0.10$, indicating that paths 1 and 3 are equally vulnerable to uncertainty and are more vulnerable than all other paths. However, when T_{cr} = 21 path 1 becomes maximally sensitive, and the project robustness is $\hat{\alpha} = \hat{\alpha}_1 = 0.17$.

Note that in all three cases considered in Table 2, the range of path-robustness values is quite large. For instance, at T_{cr} = 19, the ratio of the most to the least robust path is $\alpha_5/\alpha_3 = 9$. It is worth noting that the computations for this example take a fraction of a second on a standard personal computer. Even much larger networks are readily analyzed.

Example 2: Enhancing Reliability of Project Schedule

(Example 1 continued.) An important strategy for enhancing reliability is to reduce uncertainty. Information-gap uncertainty can be reduced by gathering information. In the context of the convex model $\mathcal{T}(\alpha)$ in (7), increased information can be expressed as reduced uncertainty weights w_n.

Fig. 2 shows the five path robustnesses versus the uncertainty weight of activity 6, with T_{cr} = 21. The plotted values of α_m at w_6 = 2 correspond to the values in the bottom row of Table 2.

From the 6th column of the participation matrix **F**, (2), it is seen that three paths depends on activity 6: paths 2, 3, and 4. In Fig. 2 it is seen that these three paths show increased robustness as w_6 decreases. However, path 1 (the lowest curve) remains constant and is always the most vulnerable to uncertainty.

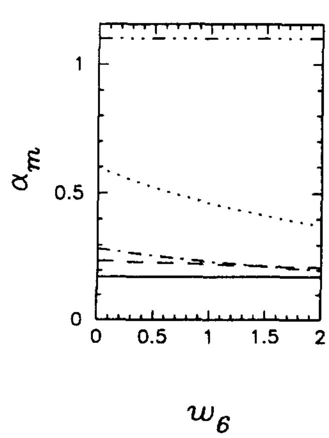

FIG. 2. α_m versus w_6, for Example 2. Symbols for Paths 1–5: (1) Solid; (2) Dashed; (3) Dot-Dash; (4) Dotted; (5) Dash-Dot-Dot-Dot

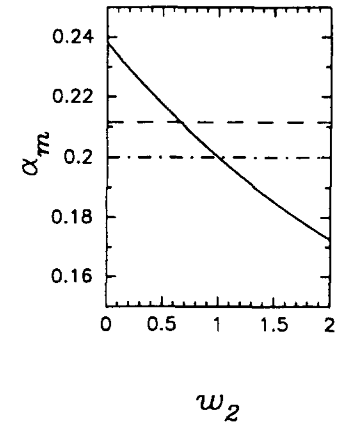

FIG. 3. α_m versus w_2, for Example 2. Symbols for Paths 1–5: (1) Solid; (2) Dashed; (3) Dot-Dash

The "critical" path, #1, can be influenced by gathering information about activity 2, whose uncertainty weight in Table 2 is w_2 = 2. In Fig. 3 it is shown that the variation of three path robustnesses versus w_2, still with T_{cr} = 21. Only the first path varies, as understood from the participation matrix, [(2)]. While the increase in the robustness of path 1 is not large, it is seen that the critical path becomes No. 3 (dot-dash) for w_2 < 1. Thus one can see that the critical path is not determined only by the nominal durations, but also by the duration uncertainties.

Example 3: Reducing Project Duration

(Example 1 continued.) In Table 3 the path-robustness calculations of Table 2 are repeated with the project data of Table 1, with the exception that now the uncertainty weights of activities 5–8, (the "core" of activity-path 3) take values different from Table 1.

From Table 3 the trade-off between reducing uncertainty on the one hand, and extending the project deadline on the other can be appreciated. Suppose, for instance, that a short-range project is being considered, in which the time overruns of the individual activities will generally be small, say on the order of no more than 10% of the nominal activity durations, this means that a project-robustness of no less than $\hat{\alpha}$ = 0.10 is needed.

Examination of the first block of numbers in Table 3, for which $w_5 = w_6 = w_7 = w_8 = 2$, shows that this requirement is satisfied with an allotted project duration of T_{cr} = 21 time units, since the most sensitive activity path has a robustness of 0.13.

TABLE 3. Path Robustnesses with Various Allotted Activity Durations for Example 3

T_{cr} (1)	α_1 (2)	α_2 (3)	α_3 (4)	α_4 (5)	α_5 (6)
		(a) $w_5 = w_6 = w_6 = w_8 = 2$			
17	0.035	0.054	**0.00**	0.11	0.70
19	0.10	0.13	**0.065**	0.22	0.90
21	0.17	0.20	**0.13**	0.33	1.10
		(b) $w_5 = w_6 = w_7 = w_8 = 1$			
17	0.035	0.061	**0.00**	0.15	0.70
19	**0.10**	0.14	0.12	0.31	0.90
21	**0.17**	0.22	0.24	0.46	1.10
		(c) $w_5 = w_6 = w_7 = w_8 = 0.5$			
17	0.035	0.066	**0.00**	0.19	0.70
19	**0.10**	0.15	0.20	0.38	0.90
21	**0.17**	0.24	0.40	0.57	1.10

However, shorter project times do not allow adequate robustness.

Now suppose uncertainty is reduced by gathering information so as to reduce the w_n, as in the second block of Table 3, for which $w_5 = w_6 = w_7 = w_8 = 1$. A project deadline of $T_{cr} = 19$ has a robustness of 10%. So a reduction in project time has been achieved in exchange for gathering information and reducing uncertainty, without diminishing project reliability.

However, further reducing the uncertainty, so that $w_5 = w_6 = w_7 = w_8 = 0.5$ (third block of Table 3), it is seen that 19 time units are still required. In fact, 17 time units will never be feasible since the nominal duration of path 3 is 17 units.

Example 4: Overlapping Project Activities

(Example 1 continued). When the allotted project duration is $T_{cr} = 21$ time units, activity-path 1 is the most vulnerable to uncertainty, as seen in Table 2: $\hat{\alpha} = \alpha_1 = 0.17$. Path 1 is the activity sequence: 1, 2, 3, 4, 16, as indicated by the first row of **F** in (2). Suppose that this activity path could be broken up by selecting another construction method enabling one to begin activity 4 when activity 3 is only partly complete. This would accelerate the path, and thereby increase the project-robustness to activity delays. So, consider the revised project schedule in Fig. 4, in which activity-path 1 is broken to form two new paths. The project schedule now has seven activity-paths:

Path 1: 1 → 2 → 3 → 16.
Path 2: 1 → 5 → 6 → 3 → 16.
Path 3: 1 → 5 → 6 → 7 → 8 → 16.
Path 4: 1 → 5 → 6 → 9 → 10 → 11 → 16.
Path 5: 1 → 12 → 13 → 14 → 15 → 16.
Path 6: 1 → 2 → 3 → 4 → 16.
Path 7: 1 → 5 → 6 → 3 → 4 → 16.

Paths 3–5 are the same as before; the other paths express the new scheduling. Instead of (2), the participation matrix for the project schedule in Fig. 4 is

$$\mathbf{F} = \begin{pmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 & 1 \\ 1 & 1 & f_{63} & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & f_{73} & 1 & 1 & 1/2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{pmatrix} \quad (16)$$

The first row has changed to represent the revised version of path 1, and the new sixth row represents path 6 in which activity 4 is initiated when activity 3 is a fraction f_{63} complete. If $f_{63} = 1$ then path 6 is identical to path 1 in the previous plan, since activity 4 begins when activity 3 ends. If $f_{63} = 0$ then activities 3 and 4 begin simultaneously. The seventh row represents path 7, and it is noted that the participation fractions f_{63} and f_{73} are in fact the same, since they both represent the fractional overlap of activities 3 and 4.

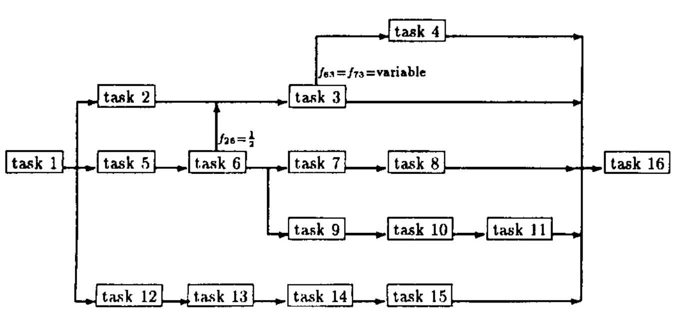

FIG. 4. Revised 16-Activity Project Schedule for Example 4

The robustnesses of the seven paths in this new schedule are shown in Table 4 for three values of the allotted project duration, with the project data of Table 1, and with $f_{63} = f_{73} = 0.5$. Note that, in comparison to Table 2, path 3 is the dominant path for all values of T_{cr} considered. [Recall that path 3 is the same in both schedules, so row 3 is the same in (2) and (16).] In other words, by overlapping activities 3 and 4 all of the paths have been "robustified" in which these activities are involved. Unlike the previous schedule, illustrated in Table 2, a shift of dominance from one path to another is no longer witnessed as the allotted duration is increased.

Table 4 is calculated with a fixed value of 0.5 for the participation fractions f_{63} and f_{73}. Now how much robustness can be achieved by adjusting the overlap of activities 3 and 6 is examined. From Table 4 it is seen that, for $T_{cr} = 19$ or 21, activity-path 6 is the next-most vulnerable path, after path 3. (Path 1 has the same robustness as path 6, but path 1 does not depend on the overlap of activities 3 and 4, so it does not vary with either f_{63} or f_{73}.) Using (14) and the definitions of $\bar{c}_m$ and f_m from (11), the robustness of the 6th activity path can be expressed as

$$\alpha_6(f_{63}) = \frac{T_{cr} - \sum_{n=1}^{N} f_{6n}\bar{t}_n}{\sum_{n=1}^{N} f_{6n}w_n\bar{t}_n} = \frac{T_{cr} - 10 - 6f_{63}}{17 + 12f_{63}} \quad (17)$$

where the project data in Table 1 have been used for the values of the uncertainty weights w_n and nominal activity durations $\bar{t}_n$.

Fig. 5 shows the value of α_6 as a function of f_{63}, for an allotted project duration of $T_{cr} = 21$ time units. The robustness of the activity path increases as f_{63} is reduced from 1 to 0. At $f_{63} = 1$ $\alpha_6 = 0.17$ is obtained, which is the same as the robustness of path 1 in the previous schedule (see Table 1). At $f_{63} = 0$ $\alpha_6 = 0.65$ is found, which is substantially more robust than the most vulnerable path, No. 3, whose robustness is also plotted and is $\alpha_3 = 0.20$. In fact, it is noted that for $f_{63} < 0.9$ path 6 is no longer the dominant path and the robustness of the entire schedule is controlled by path 3. In other words, there is nothing gained, in terms of project robustness to the

TABLE 4. Path Robustnesses with Various Allotted Project Durations for Example 4

T_{cr} (1)	α_1 (2)	α_2 (3)	α_3 (4)	α_4 (5)	α_5 (6)	α_6 (7)	α_7 (8)
17	0.17	0.23	**0.00**	0.13	0.70	0.17	0.23
19	0.26	0.33	**0.10**	0.25	0.90	0.26	0.33
21	0.35	0.43	**0.20**	0.38	1.10	0.35	0.43

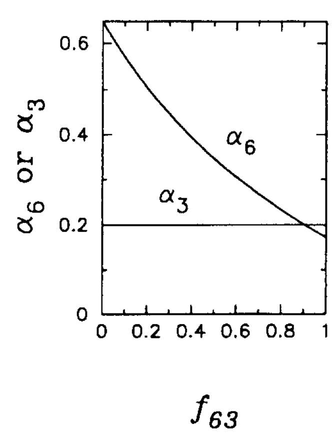

FIG. 5. α_3 and $\alpha_6(f_{63})$, for Example 4

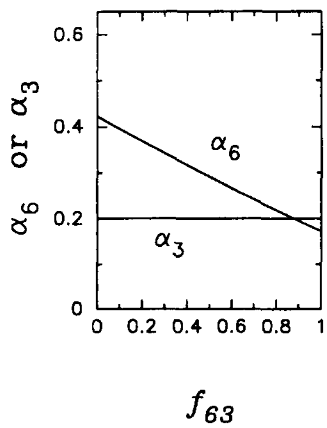

FIG. 6. α_3 and $\alpha_6(f_{63}, w_4)$, for Example 4

allotted duration, by advancing activity 4 to more than a 10% overlap with activity 3.

This analysis shows that the robustness of path 6 increases as the path is accelerated by overlapping its activities. However, often new uncertainties are introduced by overlapping activities that were previously sequential (Krishnan 1996; Laufer 1991). In the accelerated schedule of path 6, in which activities 3 and 4 overlap, the execution of activity 4 may depend on the outcome of activity 3. Since activity 4 will start before activity 3 is complete, activity 4 will suffer from increased uncertainty. This increased uncertainty in activity 4 can be expressed by increasing the uncertainty weight of activity 4 as the participation fraction f_{63} decreases. In the previous example, where activities 3 and 4 were sequential (so $f_{63} = 1$), $w_4 = 2$. Suppose that a substantial increase was anticipated in uncertainty, say $w_4 = 4$ when $f_{63} = 1/3$ (meaning that activity 4 begins when activity 3 is 1/3 complete), that $w_4 = 3$ when $f_{63} = 2/3$, and that $w_4 = 2$ when $f_{63} = 1$. For lack of more precise information, let w_4 vary linearly with f_{63} as $w_4 = -3f_{63} + 5$. Instead of (17) there is

$$\alpha_6(f_{63}, w_4) = \frac{T_{cr} - \sum_{n=1}^{N} f_{6n}\tilde{t}_n}{\sum_{n=1}^{N} f_{6n}w_n\tilde{t}_n} = \frac{T_{cr} - 10 - 6f_{63}}{11 + 12f_{63} + 3w_4} \qquad (18)$$

where w_4 varies linearly with f_{63} as indicated. Fig. 6 shows that the enhanced robustness resulting from accelerated implementation is not nearly as dramatic as in Fig. 5, due to the accompanying increase in uncertainty.

CONCLUSIONS

This paper underscores the need for higher reliability in today's projects, and presents a new concept for achieving it. The technique proposed for applying the new concept (robust reliability) meets the four characteristics of the required solution: (1) Can be applied with minimal information; (2) incorporates subjective information; (3) is simple to use and allows for easy and quick application during construction; and (4) assists in the preparation of project schedules at a desirable level of reliability.

The concept of robust reliability should help practitioners become more aware of the need to focus on the relationship between project uncertainty, project duration, and on reducing project uncertainty by collecting more information.

While master project managers do focus on the management of uncertainty, many project managers still neglect or even deny uncertainty. Many project managers can understand, visualize, and relate better to quantitative tools. Since they couldn't quantify uncertainty, they couldn't "see" it and accept it. Based on experience in attempting to change practitioners' mind-set, it is expected that the simple tool presented in this paper will contribute in two complementary ways. For those who are already aware of uncertainty and try to cope with it, it will serve as a powerful and eminently implementable tool in assessing the reliability of various project schedules, and in guiding project managers how best to enhance project reliability. For those who have ignored uncertainty, applying a quantitative technique will help realize the need to recognize and cope with it.

APPENDIX I. REFERENCES

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APPENDIX II. NOTATION

The following symbols are used in this paper:

$\mathcal{A}(T_{cr})$ = set of acceptable α-values;
c_m = duration of mth activity path;
$\bar{c}_m$ = nominal duration of mth activity path;
f_m = weighted duration of mth activity path;
f_{mn} = fractional participation of activity n in path m;
T_{cr} = allotted project duration;
$\mathcal{T}(\alpha)$ = convex model for uncertain activity durations;
t_n = duration of nth activity;
$\bar{t}_n$ = nominal duration of nth activity;
w_n = uncertainty weight of nth activity;
α = uncertainty parameter of convex model;
$\hat{\alpha}$ = robustness of project schedule; and
α_m = robustness of mth activity path.